# Program Equivalence in Sequential Core Erlang

Proving Refactoring Correctness

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# Correctness: behaviour-preservation

$e \xrightarrow{\textit{transformation}} e'$

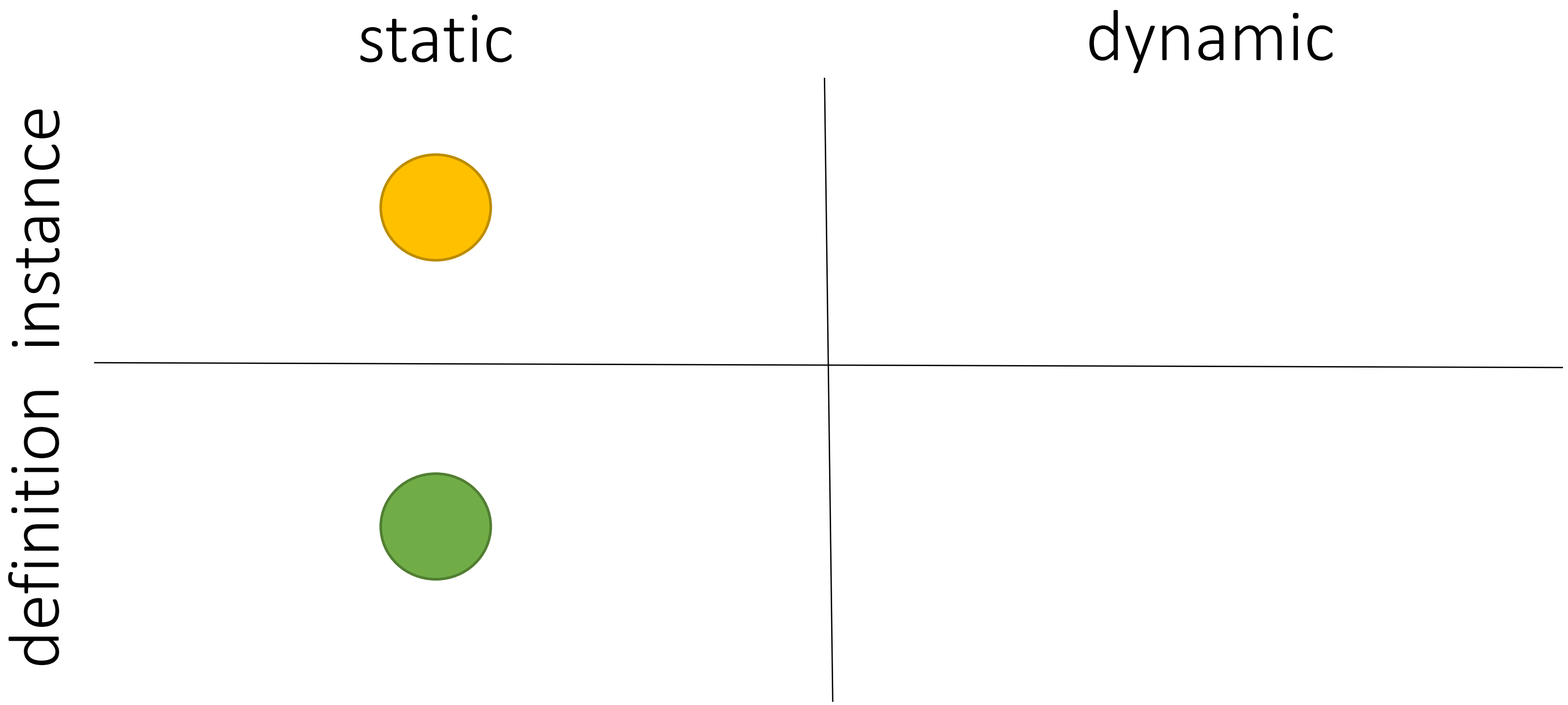
<u>*is refactoring if*</u>

$e \approx e'$

ground terms

behavioural equivalence

# Refactoring verification



# Local refactoring: eta-abstraction

$E \Rightarrow$ `(fun() ->` $E$`)()`

*is refactoring if*

`C[` $E$ `]` $\approx$ `C[ (fun() ->` $E$`)() ]`

# Local refactoring: eta-abstraction

$E \quad \Rightarrow \quad$ `(fun() -> ` $E$ `)()`

*is refactoring if*

$E \quad \approx_{ctx} \quad$ `(fun() -> ` $E$ `)()`

# Local refactoring

$$l \Rightarrow r \quad \textit{when} \quad c$$

first-order terms

*is refactoring if*

$$c \rightarrow l \approx_{\text{ctx}} r$$

contextual equivalence

# Extensive refactoring

Many refactorings are not local…

- *changing function interfaces, altering dataflow paths etc.*

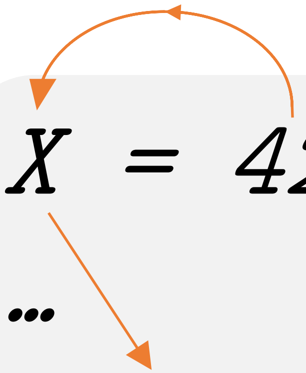
Multiple term rewrite rules applied simultaneously…

- *can we reason about rewrite strategies?*

Solution: semantic refactoring scheme

- *"semantic strategy"*

# Dataflow refactoring: eta-abstraction



```
X = 42,
…
f(X)
```

⇒

```
X = (fun() -> 42)(),
…
f(X)
```

# Dataflow refactoring: eta-abstraction

```
X = 42,
…
f(X)
```

⇒

$E \Rightarrow$ `(fun() -> E)`

```
X = fun() -> 42,
…
f(X())
```

$E \Rightarrow$ `E()`

# Dataflow refactoring: eta-abstraction

$$\mathcal{S} \Rightarrow (\texttt{fun() -> } \mathcal{S}) \quad \circ_{\texttt{df}} \quad \mathcal{R} \Rightarrow \mathcal{R}\texttt{()}$$

*is refactoring if*

$$\texttt{C}_{\texttt{df}}[\mathcal{S},\ \mathcal{R}] \approx \texttt{C}_{\texttt{df}}\ [(\texttt{fun() -> } \mathcal{S}),\ \mathcal{R}\texttt{()}]$$

# Dataflow refactoring scheme

$$S \Rightarrow \mathtt{C}_1[S] \quad \circ_{\mathtt{df}} \quad R \Rightarrow \mathtt{C}_2[R]$$

- Alter a dataflow chain of expressions
  - With a primary and a secondary rewrite rule
  - The secondary rule needs to compensate/void the primary rule
- Correctness: induction on the data flow context
  - The base cases follow from the local equivalence

$$S \approx_{\text{ctx}} \mathtt{C}_2[\mathtt{C}_1[S]]$$

# Schemes in general

$$\mathtt{C}_1[S] \Rightarrow \mathtt{C}_2[S] \quad \circ \quad \mathtt{C}_3[R] \Rightarrow \mathtt{C}_4[R]$$

- Various semantic dependencies induce refactoring schemes

- Plenty of open questions being investigated:
  - Formal definition of semantic contexts for the various schemes
  - Compound matching patterns for context-aware rewriting
  - Multiple alternative compensation rules
  - Multiple roots and loops in the dependency subgraph

By using schemes, refactoring verification boils down to $\approx$ and $\approx_{ctx}$ !

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# Motivational examples

```
mod(_, 0) -> error;
mod(N, D) -> N rem D == 0.

f() ->
  lists:foldr(fun(E, Acc) -> mod(10,E) andalso Acc end,
              true, [0,1,2,3,4,5,6]).

g() ->
  lists:foldr(fun(E, Acc) -> Acc andalso mod(10,E) end,
              true, [0,1,2,3,4,5,6]).
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```

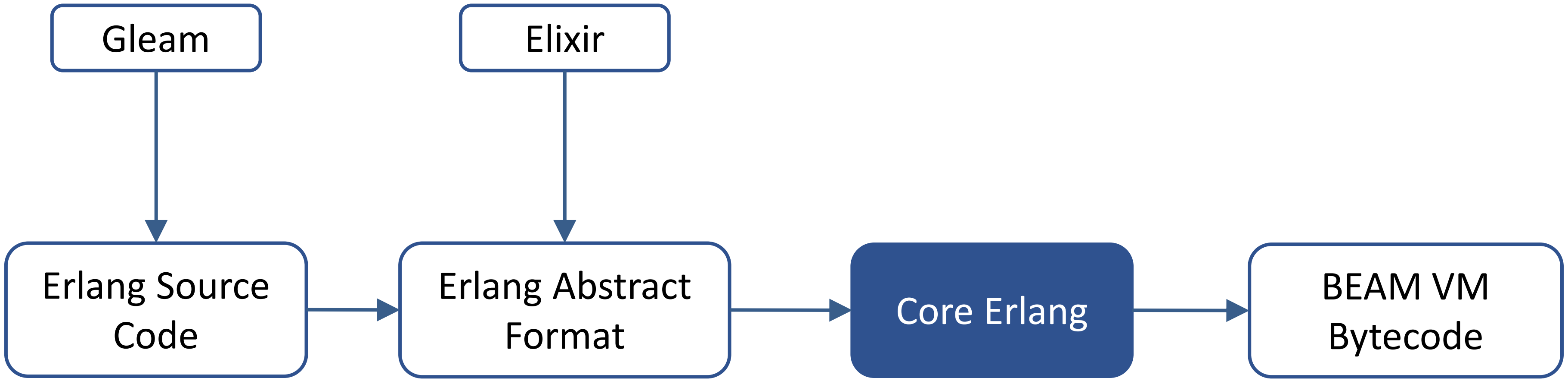
Exception

false

Is $e_1 + e_2 \approx e_2 + e_1$ for every $e_1, e_2$?
$e_1 \coloneqq$ `io:fwrite(foo)`
$e_2 \coloneqq$ `3 rem 0`

Exceptions in both cases, but side effects changed

# Motivation: Why Core Erlang?



# What is behaviour-preservation?

- Suppose, we have the semantics: $\langle \Gamma, e \rangle \Downarrow v$
- Program equivalence relations
- Congruence

Reflexive
Symmetric
Transitive

$$e_1 \approx a_1 \land e_2 \approx a_2$$

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- Congruence

$$e_1 \approx a_1 \wedge e_2 \approx a_2 \Rightarrow \textbf{do}\ e_1\ e_2 \approx \textbf{do}\ a_1\ a_2$$

- When are programs equivalent?
  - Strong equivalence

$$e \approx 2 + 2,\ e$$

  - Weak equivalence

```
io:fwrite(a), io:fwrite(b), e ≈ io:fwrite(b), io:fwrite(a), e
```

# What does it mean formally?

1st definition (based on [1]):

$$e_1 \approx e_2 \ ::= \forall(\Gamma\!: Environment)(v\!: Value)\!:$$
$$\langle \Gamma, e_1 \rangle \Downarrow v \ \Leftrightarrow \langle \Gamma, e_2 \rangle \Downarrow v$$

- ✓ Reflexive
- ✓ Symmetric
- ✓ Transitive
- ✓ Congruent

[1] Pierce, Benjamin C., et al. "Software foundations." *Webpage: http://www. cis. upenn. edu/bcpierce/sf/current/index. html* (2010).

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Major disadvantage:

$e_1 ::=$ `fun(X) -> X + 2`

$e_2 ::=$ `fun(X) -> X + 1 + 1`

What is the problem?

$$\langle\Gamma, e_1\rangle \Downarrow clos(\Gamma, [X], X + 2)$$
$$\langle\Gamma, e_2\rangle \Downarrow clos(\Gamma, [X], X + 1 + 1)$$

# Contexts

Expression context: expressions can contain ”holes”

```
do (let X = □ in X + 3) □
```

It is an inductive definition:

```
Inductive Context : Set :=
| CVar (v : Var)
| CHole
| CLet (v : Var) (c1 c2 : Context)
...
```

Substitution: $C[e]$

(`let X = □ in do (X + 3) □`)$[3+2] =$
`let X = 3 + 2 in do (X + 3) (3 + 2)`

# Contextual equivalence with equality

$$e_1 \approx_{ctx} e_2 \ ::= \forall(C{:}\,Context)(\Gamma{:}\,Environment)(v{:}\,Value){:}$$
$$\langle \Gamma, C[e_1] \rangle \Downarrow v \Leftrightarrow \langle \Gamma, C[e_2] \rangle \Downarrow v$$

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- ✓ Reflexive
- ✓ Symmetric
- ✓ Transitive
- ✓ Congruent

The previous problem still exists ☹

$e_1$ ::= `fun(X) -> X + 2`

$e_2$ ::= `fun(X) -> X + 1 + 1`

# Equivalent results

$$v_1 \approx_{val} v_2 ::= v_1 = v_2$$

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$$e_1 \approx_{ctx} e_2 ::= \forall(C{:}\,Context){:}\ \mathrm{C}[e_1] \approx_{exp} C[e_2]$$

$$e_1 \approx_{exp} e_2 ::= \forall(\Gamma{:}\,Environment)(v_1, v_2{:}\,Value){:}$$
$$\langle\Gamma, e_1\rangle \Downarrow v_1 \wedge \langle\Gamma, e_2\rangle \Downarrow v_2 \Rightarrow v_1 \approx_{val} v_2$$

$$v_1 \approx_{val} v_2 ::= v_1 = v_2$$
$$clos(\Gamma, [x_1, \dots, x_n], e_1) \approx_{val} clos(\Gamma', [x_1, \dots, x_n], e_2) ::=$$
$$\forall v_1, \dots, v_n{:}\,(fun(x_1, \dots, x_n) \rightarrow e_1)(v_1, \dots, v_n) \approx_{exp}$$
$$(fun(x_1, \dots, x_n) \rightarrow e_2)(v_1, \dots, v_n)$$

- ✓ Reflexive
- ✓ Symmetric
- ✓ Transitive
- x Congruent

[2] Owens, Scott, et al. "Functional big-step semantics." *European Symposium on Programming*. Springer, Berlin, Heidelberg, 2016.
[3] Pitts, Andrew M. "Operationally-based theories of program equivalence." *Semantics and Logics of Computation* 14 (1997): 241.

# Congruence: application?

- Goal: $C_0(C_1, \ldots, C_n)[e_1] \approx_{exp}$
  $C_0(C_1, \ldots, C_n)[e_2]$

# Congruence: application?

- Goal: $C_0(C_1, \dots, C_n)[e_1] = C_0[e_1](C_1[e_1], \dots, C_n[e_2]) \approx_{exp}$
  $C_0(C_1, \dots, C_n)[e_2] = C_0[e_2](C_1[e_2], \dots, C_n[e_2])$

- Induction hypothesis: $\forall (e_1, e_2 : Exp) : e_1 \approx_{exp} e_2 \rightarrow C_i[e_1] \approx_{exp} C_i[e_2]$

- If $C_0[e_1] \Downarrow clos(\Gamma, [x_1, \dots, x_n], b_1)$ and $C_0[e_2] \Downarrow clos(\Gamma', [x_1, \dots, x_n], b_2)$ then:

$$\forall v_1, \dots, v_n : (fun(x_1, \dots, x_n) \rightarrow b_1)(v_1, \dots, v_n) \approx_{exp}$$
$$(fun(x_1, \dots, x_n) \rightarrow b_2)(v_1, \dots, v_n)$$

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- Goal: $C_0(C_1, \dots, C_n)[e_1] = C_0[e_1](C_1[e_1], \dots, C_n[e_2]) \approx_{exp}$
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- Induction hypothesis: $\forall(e_1, e_2 : Exp) : e_1 \approx_{exp} e_2 \rightarrow C_i[e_1] \approx_{exp} C_i[e_2]$

- If $C_0[e_1] \Downarrow clos(\Gamma, [x_1, \dots, x_n], b_1)$ and $C_0[e_2] \Downarrow clos(\Gamma', [x_1, \dots, x_n], b_2)$ then:

$$\forall v_1, \dots, v_n : (fun(x_1, \dots, x_n) \rightarrow b_1)(v_1, \dots, v_n) \approx_{exp} (fun(x_1, \dots, x_n) \rightarrow b_2)(v_1, \dots, v_n)$$

- However, we only have equivalent parameters

# Back to the definition

$$e_1 \approx_{ctx} e_2 \ ::= \forall(C{:}\,Context){:}\ \mathrm{C}[e_1] \approx_{exp} C[e_2]$$

$$e_1 \approx_{exp} e_2 \ ::= \forall(\Gamma{:}\,Environment)(v_1, v_2{:}\,Value){:}$$
$$\langle \Gamma, e_1 \rangle \Downarrow v_1 \wedge \langle \Gamma, e_2 \rangle \Downarrow v_2 \Rightarrow v_1 \approx_{val} v_2$$

$$v_1 \approx_{val} v_2 ::= v_1 = v_2$$
$$clos(\Gamma, [x_1, \dots, x_n], e_1) \approx_{val} clos(\Gamma', [x_1, \dots, x_n], e_2) ::=$$
$$\forall v_1, \dots, v_n{:}\ (fun(x_1, \dots, x_n) \rightarrow e_1)(v_1, \dots, v_n) \approx_{exp}$$
$$(fun(x_1, \dots, x_n) \rightarrow e_2)(v_1, \dots, v_n)$$

Same environment?
Same parameters?

- ✓ Reflexive
- ✓ Symmetric
- ✓ Transitive
- x Congruent

# Let us fix the congruence premises

$$v_1 \approx_{val} v_2 ::= v_1 = v_2$$
$$clos(\Gamma, [x_1, \dots, x_n], e_1) \approx_{val} clos(\Gamma', [x_1, \dots, x_n], e_2) ::=$$
$$\forall v_1, \dots, v_n, v_1', \dots, v_n' \colon v_1 \approx_{val} v_1' \land \cdots \land v_n \approx_{val} v_n' \Rightarrow$$
$$\Gamma[x_1 \leftarrow v_1, \dots, x_n \leftarrow v_n], e_1 \approx_{exp} \Gamma'[x_1 \leftarrow v_1', \dots, x_n \leftarrow v_n'], e_2$$

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$$v_1 \approx_{val} v_2 ::= v_1 = v_2$$
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$$\forall v_1, \ldots, v_n, v_1', \ldots, v_n' \colon v_1 \approx_{val} v_1' \land \cdots \land v_n \approx_{val} v_n' \Rightarrow$$
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$$\forall v_1, \dots, v_n, v'_1, \dots, v'_n\colon v_1 \approx_{val} v'_1 \land \cdots \land v_n \approx_{val} v'_n \Rightarrow$$
$$\Gamma[x_1 \leftarrow v_1, \dots, x_n \leftarrow v_n], e_1 \approx_{exp} \Gamma'[x_1 \leftarrow v'_1, \dots, x_n \leftarrow v'_n], e_2$$

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$$\Gamma, e_1 \approx_{exp} \Gamma', e_2 \ ::= \forall (v_1, v_2 : Value):$$
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$$v_1 \approx_{val} v_2 ::= v_1 = v_2$$
$$clos(\Gamma, [x_1, \ldots, x_n], e_1) \approx_{val} clos(\Gamma', [x_1, \ldots, x_n], e_2) ::=$$
$$\forall v_1, \ldots, v_n, v_1', \ldots, v_n' : v_1 \approx_{val} v_1' \wedge \cdots \wedge v_n \approx_{val} v_n' \Rightarrow$$
$$\Gamma[x_1 \leftarrow v_1, \ldots, x_n \leftarrow v_n], e_1 \approx_{exp} \Gamma'[x_1 \leftarrow v_1', \ldots, x_n \leftarrow v_n'], e_2$$

Using this: $\Gamma, 0 \approx_{exp} \Gamma,$ `letrec 'f'/0 = fun() -> apply 'f'/0 in apply 'f'/0()`

# Adding termination criteria

$$\Gamma, e_1 \approx_{exp} \Gamma', e_2 \; ::= \forall (v_1, v_2 : Value):$$
$$\langle \Gamma, e_1 \rangle \Downarrow \Leftrightarrow \langle \Gamma', e_2 \rangle \Downarrow \land$$
$$(\langle \Gamma, e_1 \rangle \Downarrow v_1 \land \langle \Gamma', e_2 \rangle \Downarrow v_2 \Rightarrow v_1 \approx_{val} v_2)$$

$$v_1 \approx_{val} v_2 ::= v_1 = v_2$$
$$clos(\Gamma, [x_1, \dots, x_n], e_1) \approx_{val} clos(\Gamma', [x_1, \dots, x_n], e_2) ::=$$
$$\forall v_1, \dots, v_n, v_1', \dots, v_n' : v_1 \approx_{val} v_1' \land \cdots \land v_n \approx_{val} v_n' \Rightarrow$$
$$\Gamma[x_1 \leftarrow v_1, \dots, x_n \leftarrow v_n], e_1 \approx_{exp} \Gamma'[x_1 \leftarrow v_1' \dots x_n \leftarrow v_n'], e_2$$

Work in progress
- ✓ Reflexive
- Symmetric
- Transitive
- Congruent

# Examples

$$e_1 \approx e_2 \ ::= \forall(\Gamma: Environment): \Gamma, e_1 \approx_{exp} \ \Gamma, e_2$$

```
1 ≈ 0 + 1
```

```
sum(2) ≈ 3
```

```
fun() -> 1 ≈ fun() -> 0 + 1
```

```
fun(X) -> e ≈ fun(X) -> (fun(X) -> e)(X)
```

```
0 ≉ letrec 'f'/0 = fun() -> apply 'f'/0 in apply 'f'/0()
```

# Coq embedding

- We have: Core Erlang semantics
- The relations above are not accepted by the positivity checker
- Workaround [4]
  - Define the parts of the relation piece-by-piece
  - Assemble the relation with a Fixpoint
  - However, it recurses over the type : there is no typing in Erlang/Core Erlang
  - Solution: use the size of the terms instead

[4] Culpepper R., Cobb A. (2017) Contextual Equivalence for Probabilistic Programs with Continuous Random Variables and Scoring. In: Yang H. (eds) Programming Languages and Systems. ESOP 2017. Lecture Notes in Computer Science, vol 10201. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-662-54434-1_14

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# GitHub

*https://github.com/harp-project*

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