

# Why Functional Programming Matters



John Hughes & Mary Sheeran

QuviQ

**CHALMERS**

# Functional Programming à la 1940s

- Minimalist: who needs booleans?
- A boolean just *makes a choice!*

**true**    **x y = x**

**false** **x y = y**

- We can *define* if-then-else!

**ifte bool t e =**  
**bool t e**

# Who needs integers?

- A (positive) integer just *counts loop iterations!*

**two**    **f** **x** = **f** (**f** **x**)

**one**    **f** **x** = **f** **x**

**zero** **f** **x** = **x**

- To recover a "normal" integer...

```
*Church> two (+1) 0  
2
```

# Look, Ma, we can add!

- Addition by *sequencing* loops

$$\text{add } m \ n \ f \ x = m \ f \ (n \ f \ x) \quad \left. \begin{array}{l} \text{ } \\ \text{ } \end{array} \right\} \begin{array}{l} m \\ n \end{array}$$

- Multiplication by *nesting* loops!

$$\text{mul } m \ n \ f \ x = m \ (n \ f) \ x \quad \left. \begin{array}{l} \text{ } \\ \text{ } \end{array} \right\} \begin{array}{l} n \\ m \end{array}$$

```
*Church> add one (mul two two) (+1) 0  
5
```

# Factorial à la 1940

```
fact n =  
  ifte (iszero n)  
    one  
    (mul n (fact (decr n)))
```

```
*Church> fact (add one (mul two two)) (+1) 0  
120
```

# A couple more auxiliaries

- Testing for zero

```
iszero n =  
  n (\_ -> false) true
```

- Decrementing...

```
decr n =  
  n (\m f x-> f (m incr zero))  
    zero  
    (\x->x)  
    zero
```

Booleans, integers, (and other data structures) *can be entirely replaced by functions!*

*"Church encodings"*

Early versions of the Glasgow Haskell compiler actually implemented data-structures this way!



*Alonzo Church*

# Before you try this at home...

# Church.hs:27:35:

Occurs check: cannot construct the infinite type:

$$t \sim t \rightarrow t \rightarrow t$$

Actual type:

Expected type:

[illegible][illegible]



But wait, there's more...

## Relevant bindings include

[illegible][illegible]

```

-> (((t -> t -> t) -> t -> t) -> (t -> t -> t) -> t -> t -> t)
-> (((t -> t -> t) -> t -> t) -> (t -> t -> t) -> t -> t -> t)
-> ((t -> t -> t) -> t -> t)
-> (t -> t -> t)
-> t
-> t
-> t)
-> (((((t -> t -> t) -> t -> t) -> (t -> t -> t) -> t -> t -> t)
-> ((t -> t -> t) -> t -> t) -> (t -> t -> t) -> t -> t -> t)
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-> (((t -> t -> t) -> t -> t) -> (t -> t -> t) -> t -> t -> t)
-> (((t -> t -> t) -> t -> t) -> (t -> t -> t) -> t -> t -> t)
-> ((t -> t -> t) -> t -> t)
-> (t -> t -> t)
-> t
-> t
-> t)
-> t
-> t)

```

(bound at Church.hs:27:1)

In the first argument of 'mul', namely 'n'

In the third argument of 'ifte', namely '(mul n (fact (decr n)))'

The type-checker needs a *little bit* of help

```
fact ::  
  (forall a. (a->a) ->a->a) ->  
  (a->a) -> a -> a
```

# Factorial à la 1960



```
(LABEL FACT (LAMBDA (N)
  (COND ((ZEROP N) 1)
        (T (TIMES N (FACT (SUB1 N)))))))
```

**Higher-order functions!**

```
(MAPLIST FACT (QUOTE (1 2 3 4 5)))

(1 2 6 24 120)
```

# The Next 700 Programming Languages

P. J. Landin

*Univac Division of Sperry Rand Corp., New York, New York*

“... today ... 1,700 special programming languages used to ‘communicate’ in over 700 application areas.”—*Computer Software Issues*, an American Mathematical Association Prospectus, July 1965.



## Factorial in ISWIM

**fac(5)**

**where rec fac(n) =**

**(n=1) → 1;**

**n\*fac(n-1)**

# Laws

$(\text{MAPLIST } F (\text{REVERSE } L)) \equiv (\text{REVERSE } (\text{MAPLIST } F L))$

What's the point of two different ways to do the same thing?

Wouldn't *two* facilities be better than one?

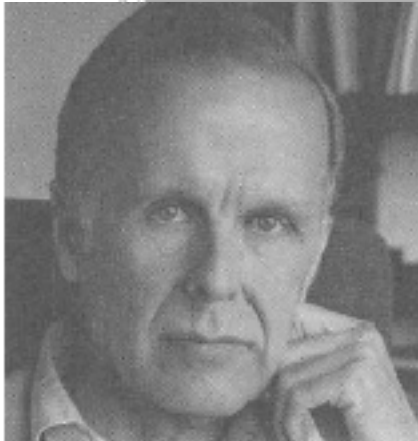
**Expressive power should be by design, rather than by accident!**



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# Can Programming Be Liberated from the von Neumann Style? A Functional Style and Its Algebra of Programs

John Backus  
IBM Research Laboratory, San Jose



Turing award 1977

[Paper 1978](#)

**Conventional programming  
languages are growing ever  
more enormous,  
but not stronger.**

**Inherent defects at the most  
basic level cause them to be  
both **fat** and **weak**:**



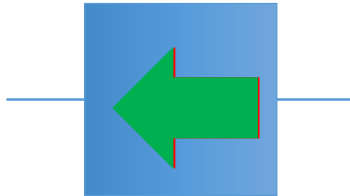
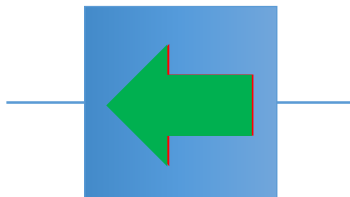
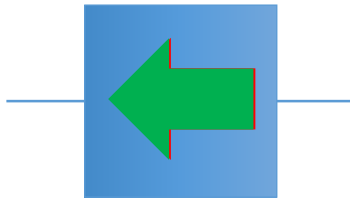
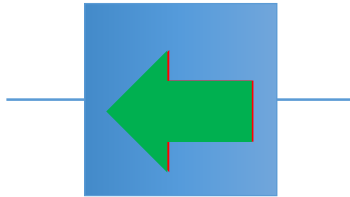
Word-at-a-time



their inability to effectively use  
powerful combining forms  
for building new programs from  
existing ones

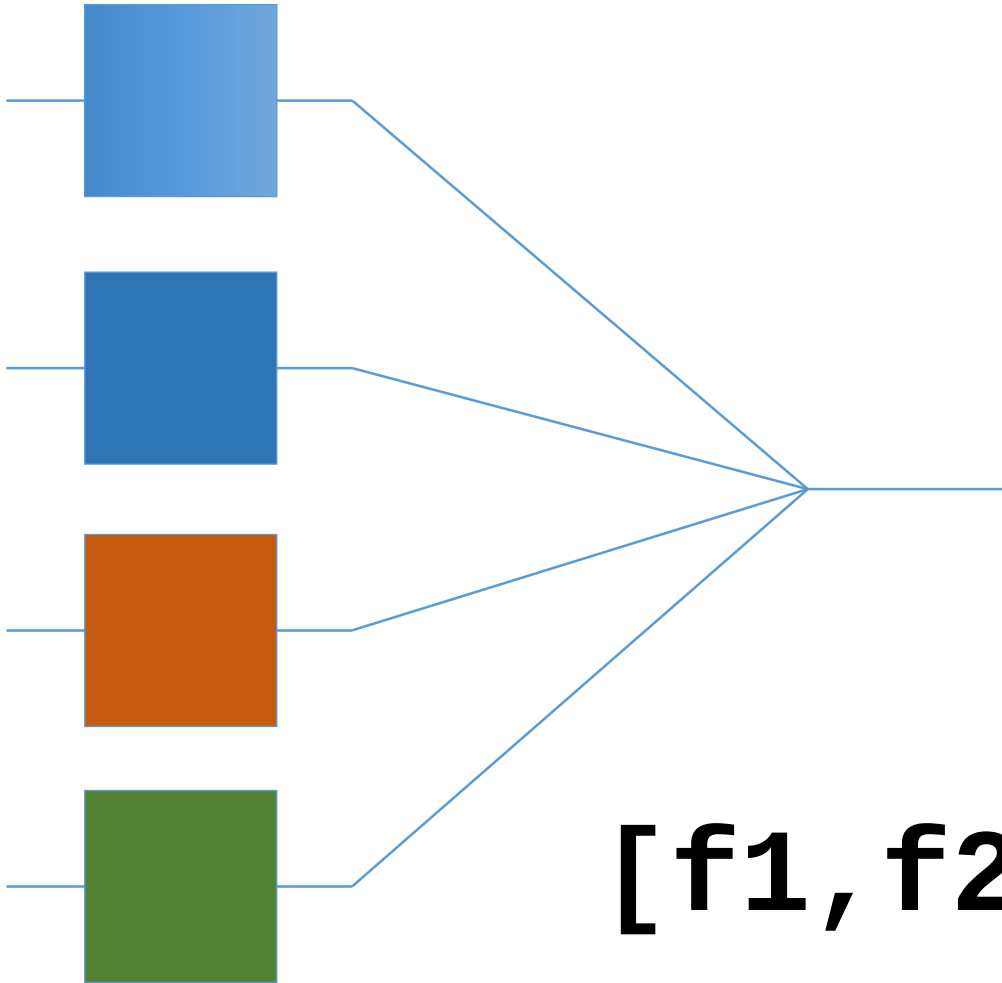


apply to all



$\alpha f$

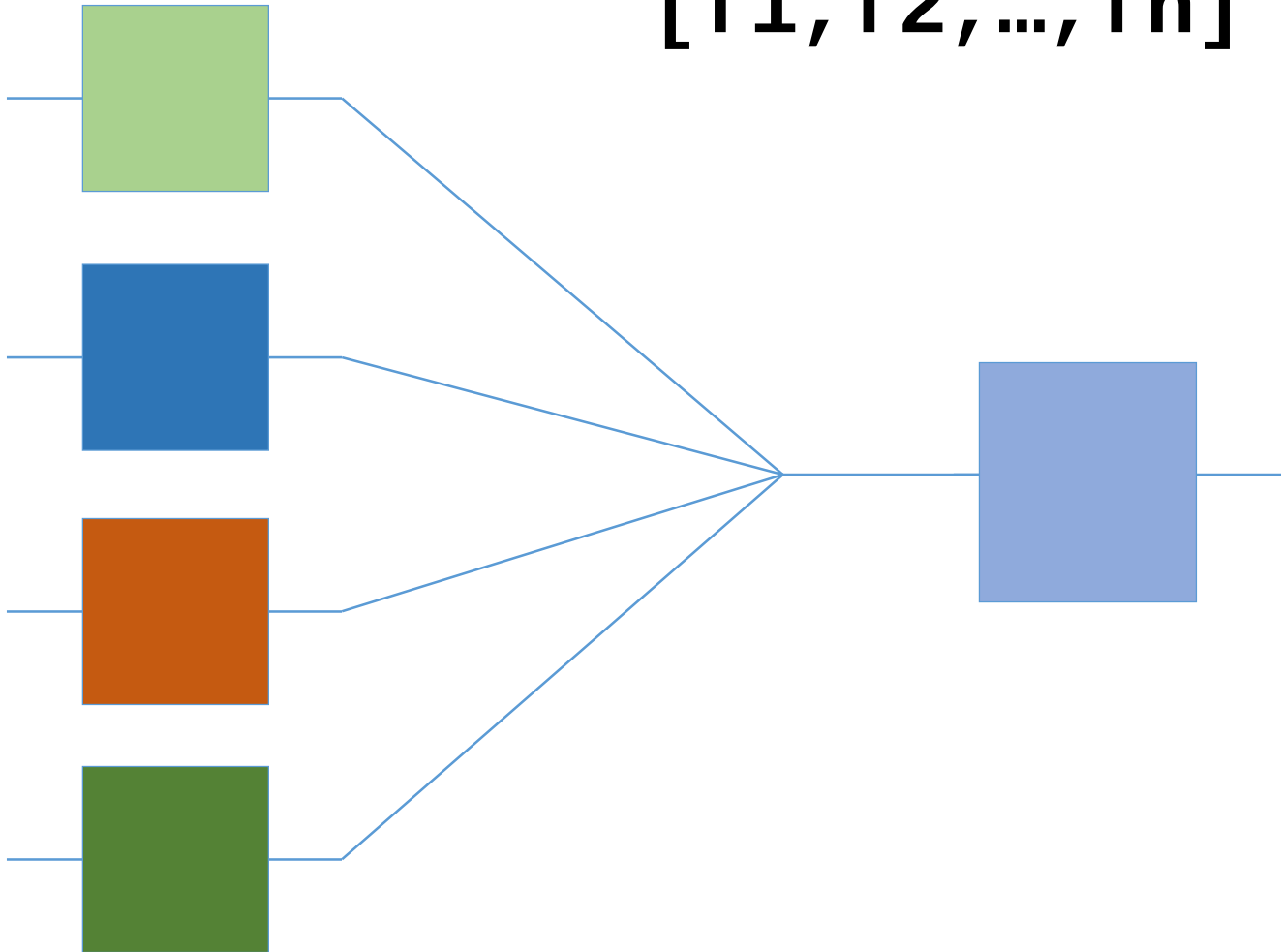
construction

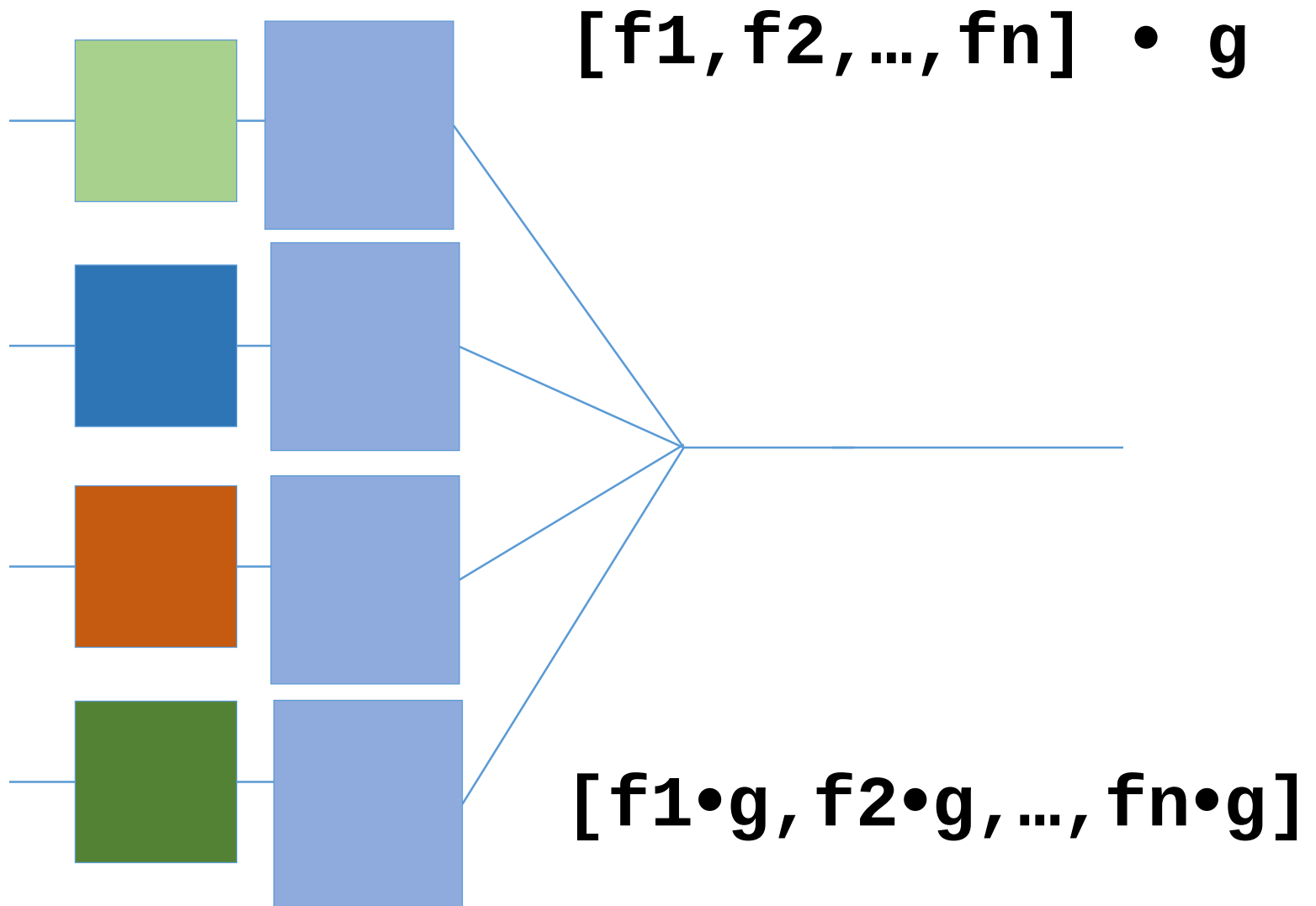


**[f1, f2, f3, f4]**

**their lack of useful  
mathematical properties for  
reasoning about programs**

**$[f_1, f_2, \dots, f_n] \cdot g$**





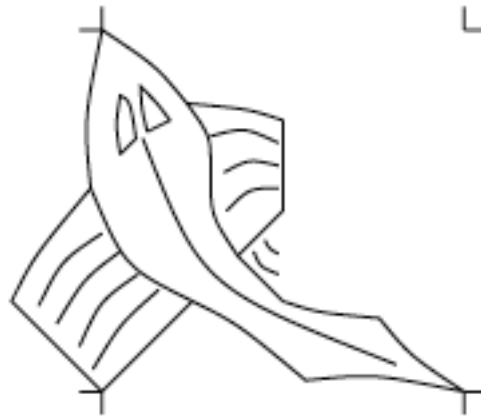


```
c := 0;  
for i := 1 step 1 until n do  
  c := c + a[i] × b[i]
```

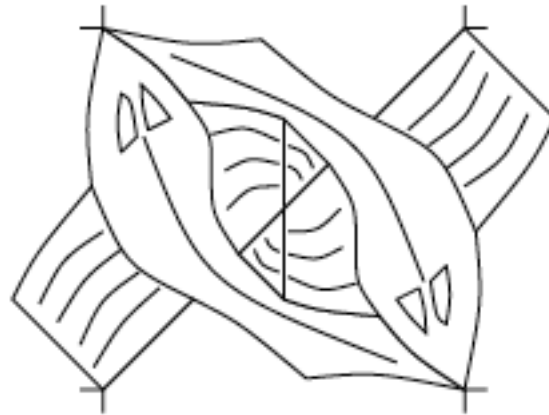
**Def IP = (/ +) • (α ×) • Trans**

*Peter Henderson, Functional Geometry, 1982*

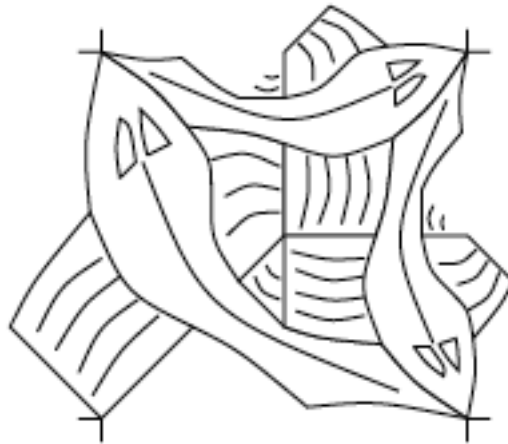




**fish**



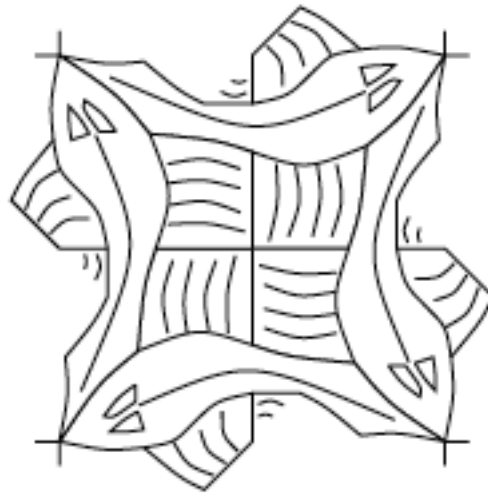
over (fish, rot (rot (fish)))



```
t = over (fish, over (fish2, fish3))
```

```
fish2 = flip (rot45 fish)
```

```
fish3 = rot (rot (rot (fish2)))
```

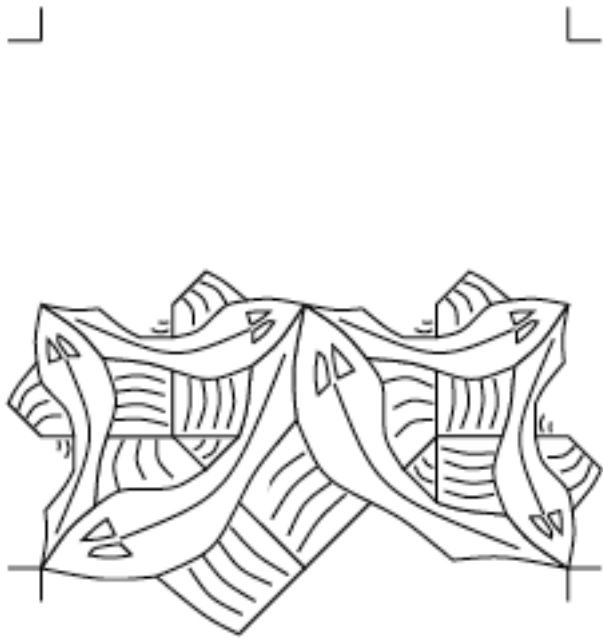


```
u = over (over (fish2, rot (fish2)),  
          over (rot (rot (fish2)),  
                rot (rot (rot (fish2))))))
```

|   |   |
|---|---|
| P | Q |
| R | S |

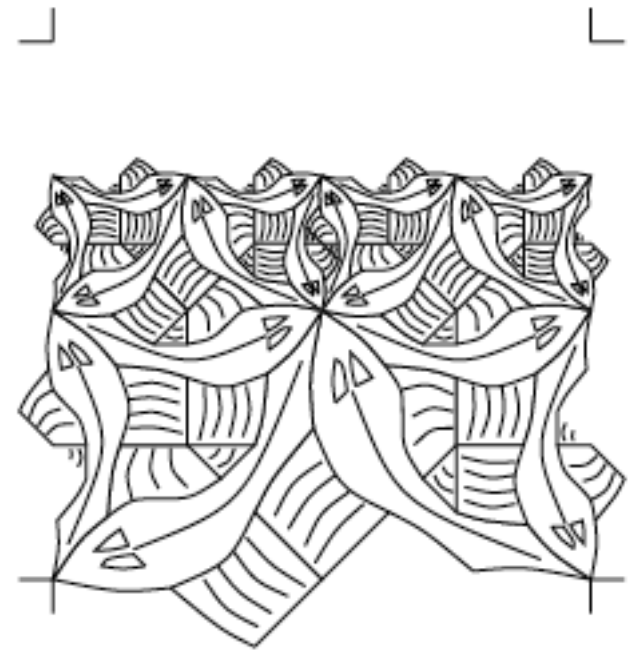
quartet





`quartet(nil, nil,  
rot(t), t)`

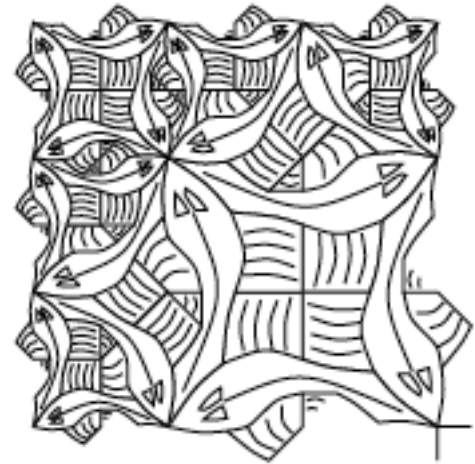
`side1`



`quartet(side1, side1,  
rot(t), t )`

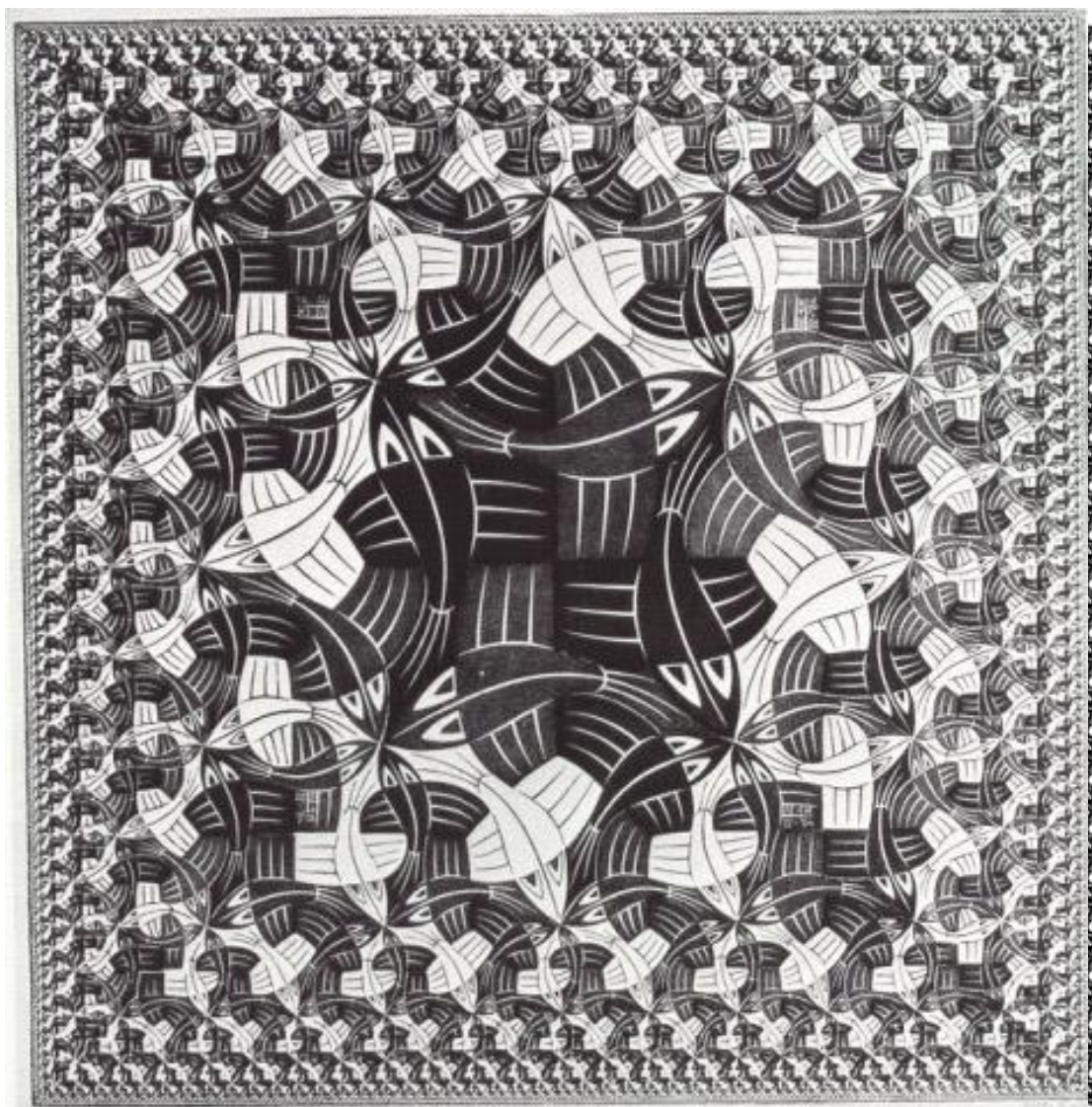


quartet (nil,nil,nil,u)  
corner1



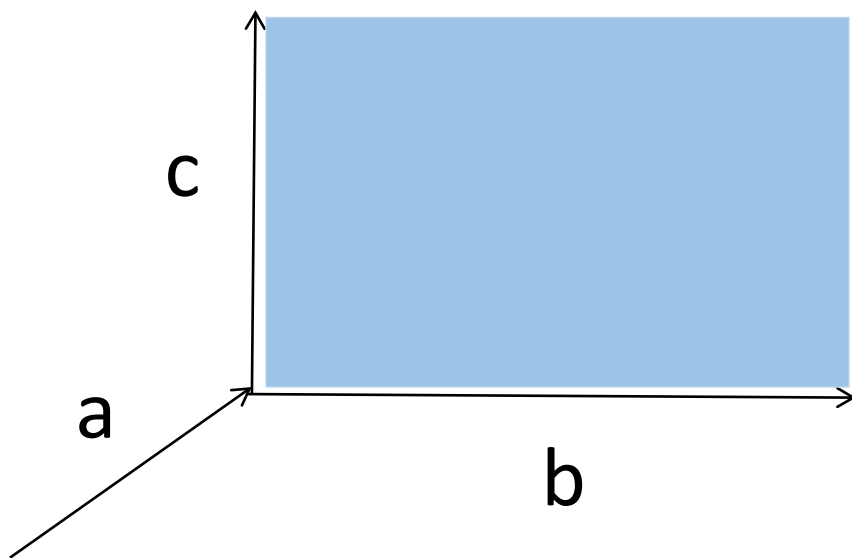
quartet(corner1,  
side1,  
rot(side1),  
u)

```
squarelimit = nonet(  
    corner,      side,      rot(rot(rot(corner))),  
    rot(side),   u,         rot(rot(rot(side))),  
    rot(corner), rot(rot(side)), rot(rot(corner)))
```

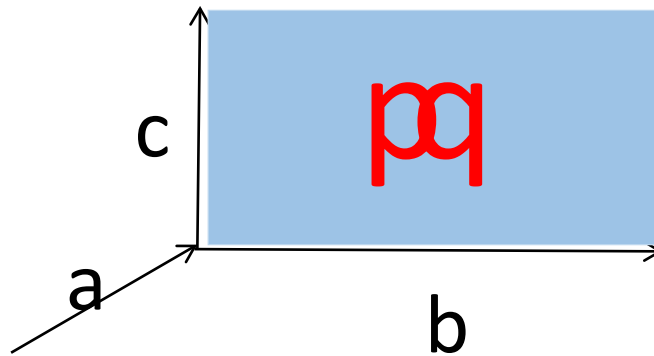


picture = function

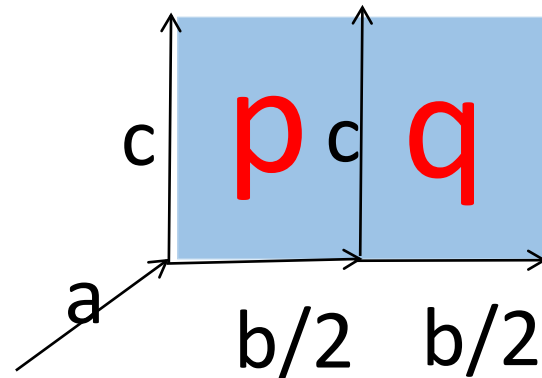
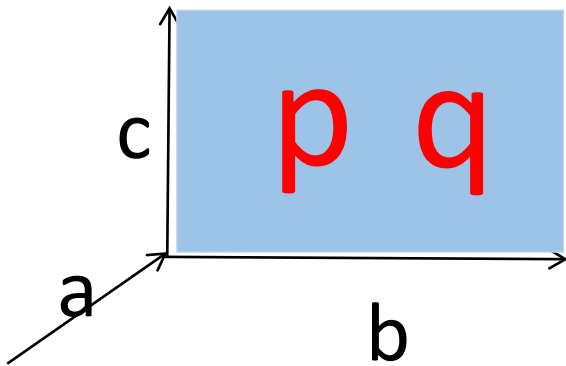
picture = function



$$\text{over } (p, q) \ (a, b, c) = \\ p(a, b, c) \cup q(a, b, c)$$

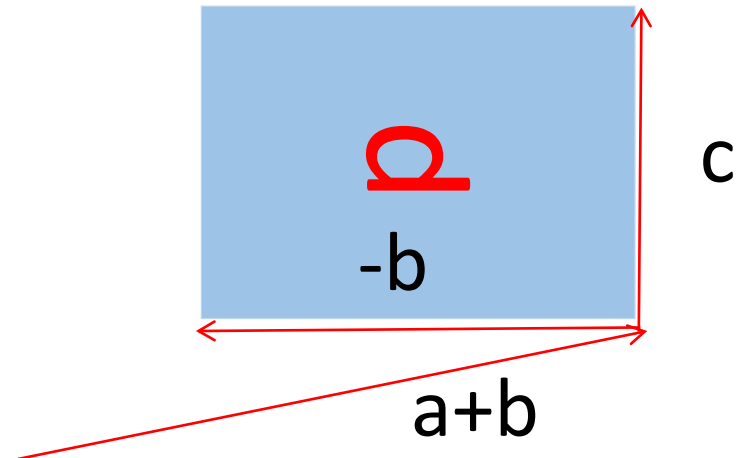
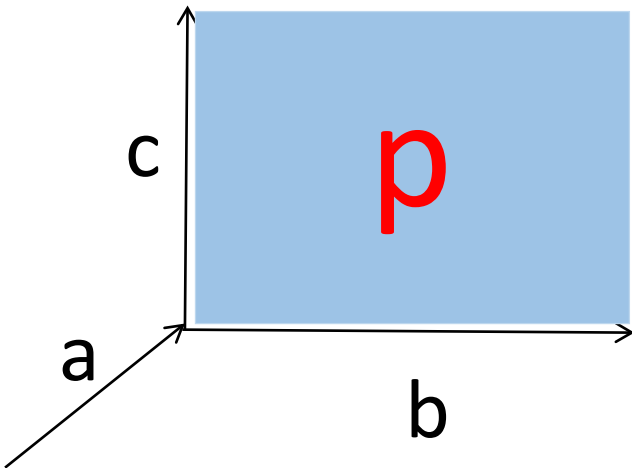


$$\text{beside } (p, q) \ (a, b, c) = \\ p(a, b/2, c) \cup q(a+b/2, b/2, c)$$





$$\text{rot}(p) \ (a, b, c) = p(a+b, c, -b)$$



# Laws

$$\begin{aligned} &\text{rot}(\text{above}(p, q)) \\ &= \\ &\text{beside}(\text{rot}(p), \text{rot}(q)) \end{aligned}$$



It seems there is a positive correlation between the simplicity of the rules and the quality of the algebra as a description tool.

Whole values

Combining forms

Algebra as litmus test

Functions as representations

# Haskell vs. Ada vs. C++ vs. Awk vs. ... An Experiment in Software Prototyping Productivity\*

Paul Hudak  
Mark P. Jones

Yale University  
Department of Computer Science  
New Haven, CT 06518  
`{hudak-paul, jones-mark}@cs.yale.edu`



July 4, 1994



Time 40.0:

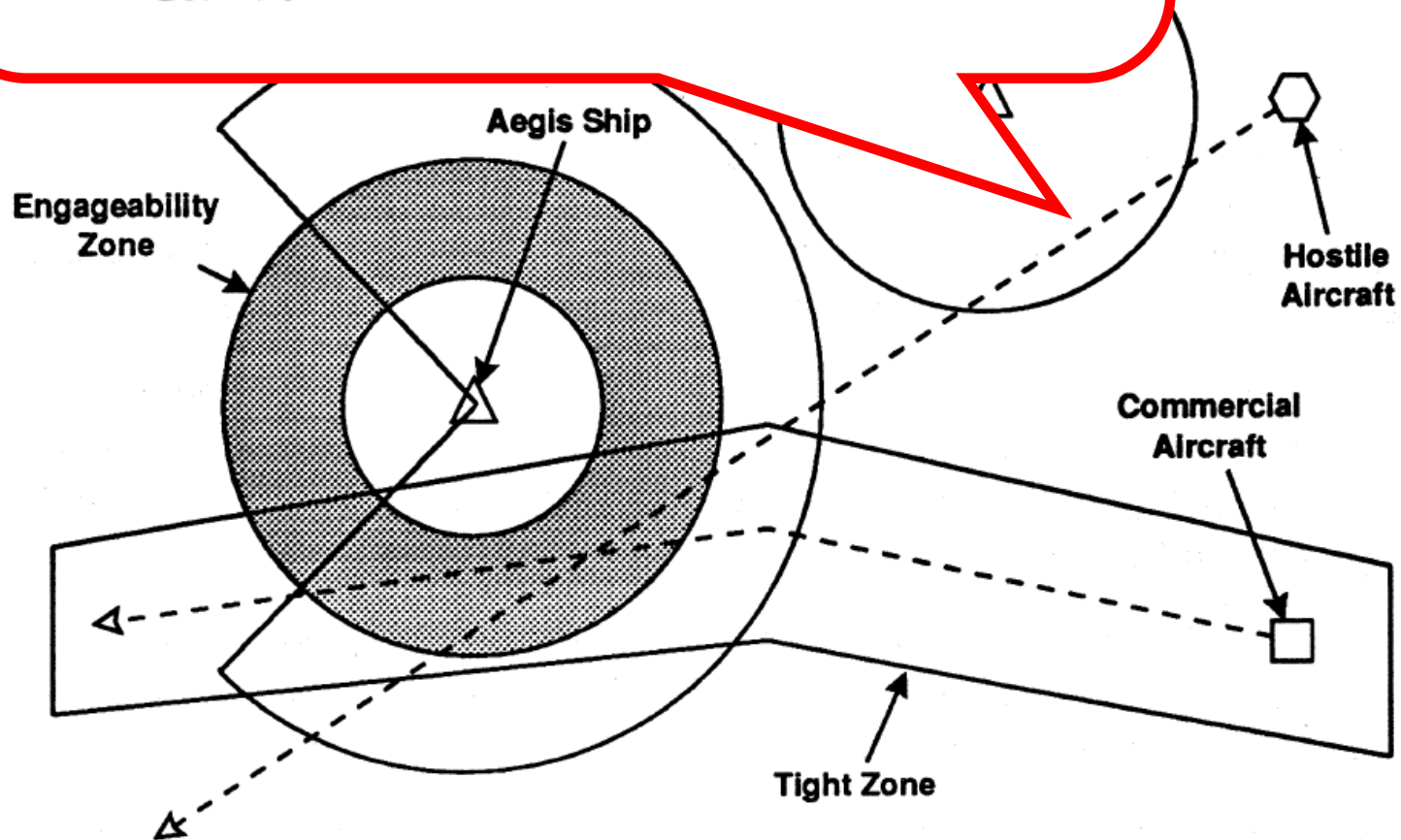
commercial aircraft: (100.0,43.0)

-- In engageability zone

-- In tight zone

hostile craft: (210.0,136.0)

-- In carrier slave doctrine



# Functions as Data

```
> type Region    =    Point -> Bool
```

Including 29 lines of  
inferable type  
signatures/synonyms

A student, given 8 days  
to learn Haskell, w/o  
knowledge of Yale group

| Language            | Lines of code | Lines of document | Development time (hours) |
|---------------------|---------------|-------------------|--------------------------|
| (1) Haskell         | 85            | 465               | 10                       |
| (2) Ada             | 767           | 7                 | 23                       |
| (3) Ada9X           | 800           |                   | 28                       |
| (4) C++             | 1105          | 130               | -                        |
| (5) Awk/Nawk        | 250           | 150               | -                        |
| (6) Rapide          | 157           | 0                 | 54                       |
| (7) Griffin         | 251           | 0                 | 34                       |
| (8) Proteus         | 293           | 79                | 26                       |
| (9) Relational Lisp | 274           | 12                | 3                        |
| (10) Haskell        | 156           | 112               | 8                        |

Figure 3: Summary of Prototype Software Development Metrics

# Reaction...

”too cute for its own good”

...higher-order functions just a trick, probably not useful in other contexts



# Lazy Evaluation (1976)



Henderson and Morris  
*A lazy evaluator*



Friedman and Wise  
*CONS should not evaluate  
its arguments*

# “The Whole Value” can be $\infty$ !

- The *infinite list* of natural numbers  
**[0, 1, 2, 3 ...]**

*Consumer* decides  
how many to  
compute

- All the iterations of a function  
**iterate f x = [x, f x, f (f x), ...]**
- A consumer for numerical methods  
**limit eps xs =**  
*<first element of **xs** within **eps** of its predecessor>*

# Some numerical algorithms

- Newton-Raphson square root

```
sqrt a = limit eps (iterate next 1.0)  
where next x = (x + a/x) / 2
```

- Derivatives

```
deriv f x =  
  limit eps (map slope (iterate (/2) 1.0))  
where slope h = (f (x+h) - f x) / h
```

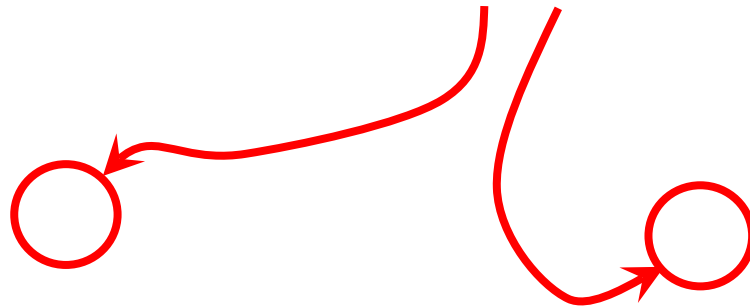
[1, 1/2, 1/4, 1/8...]

*Same convergence check*

*Different approximation sequences*

# Speeding up convergence

The smaller  $h$  is, the better the approximation



Differentiation

Integration

The right  
answer

$$A + B * h^n$$

An error term

# Eliminating the error term

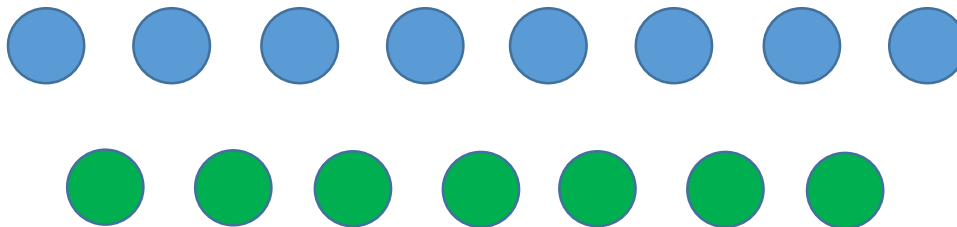
- Given:

$$A + B * h^n$$

$$A + B * (h/2)^n$$

Two successive approximations

- Solve for A and B!



**improve**  $n$  **xs** *converges faster than xs*

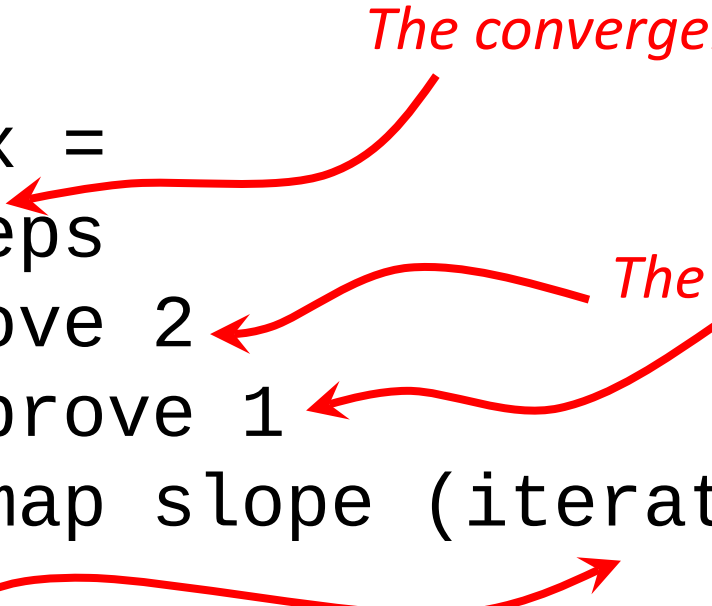
# Really fast derivative

```
deriv f x =  
  limit eps  
    (improve 2  
      (improve 1  
        (map slope (iterate (/2) 1.0))))))
```

*The convergence check*

*The improvements*

*The approximations*



Everything is programmed *separately* and easy to understand—thanks to “whole value programming”

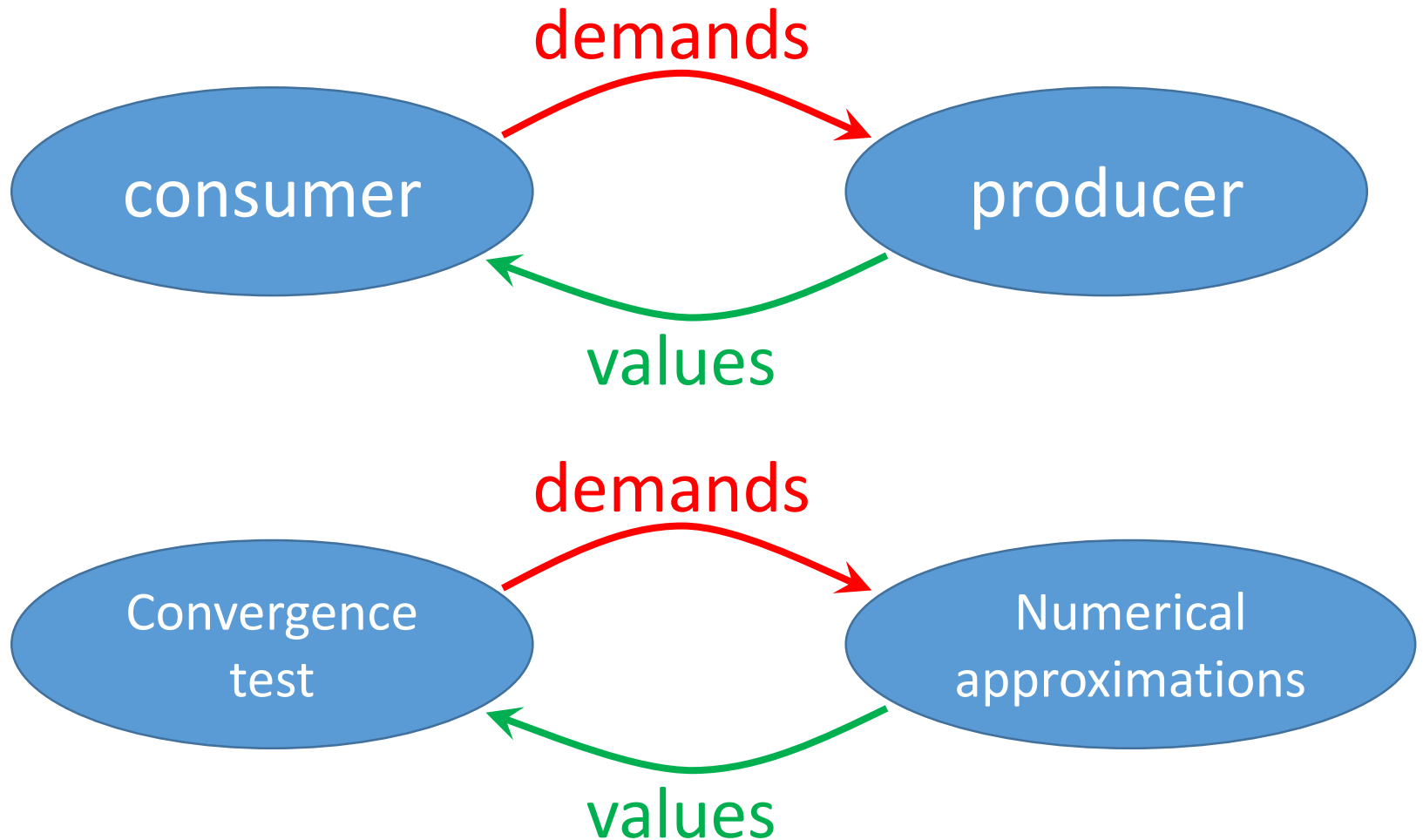
# Why Functional Programming Matters

John Hughes  
The University, Glasgow



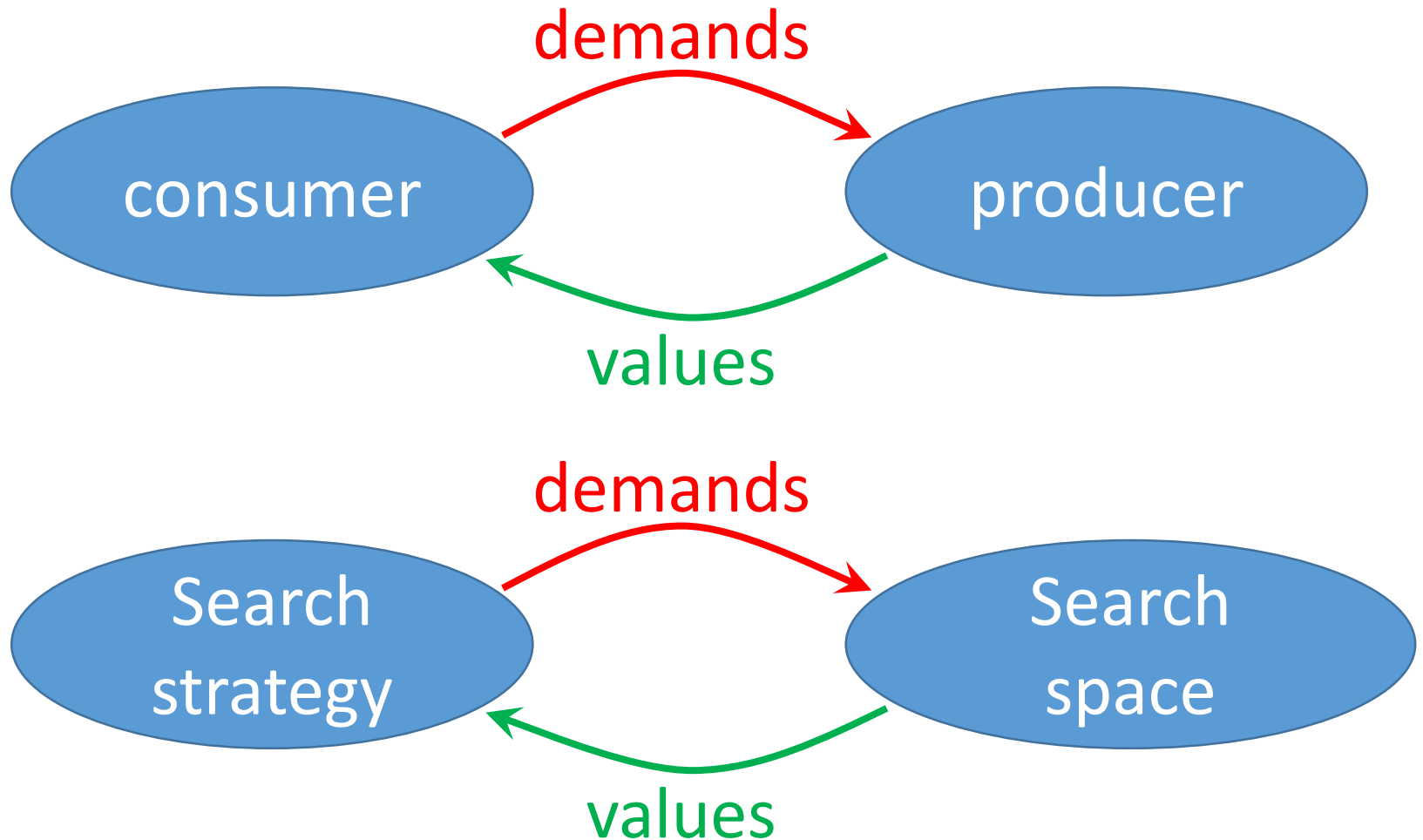
1990

# Lazy producer-consumer



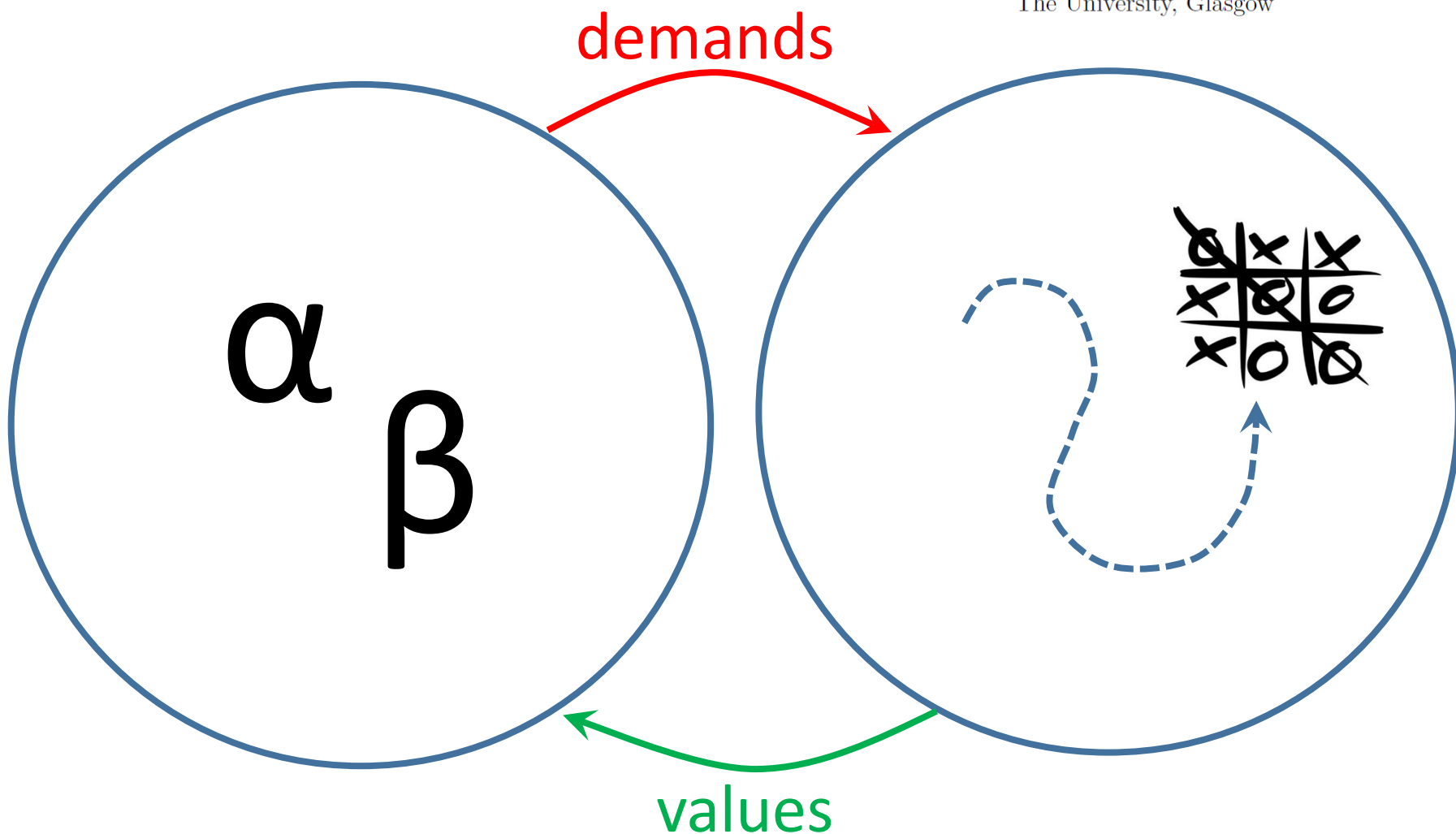


# Lazy producer-consumer



# Why Functional Programming Matters

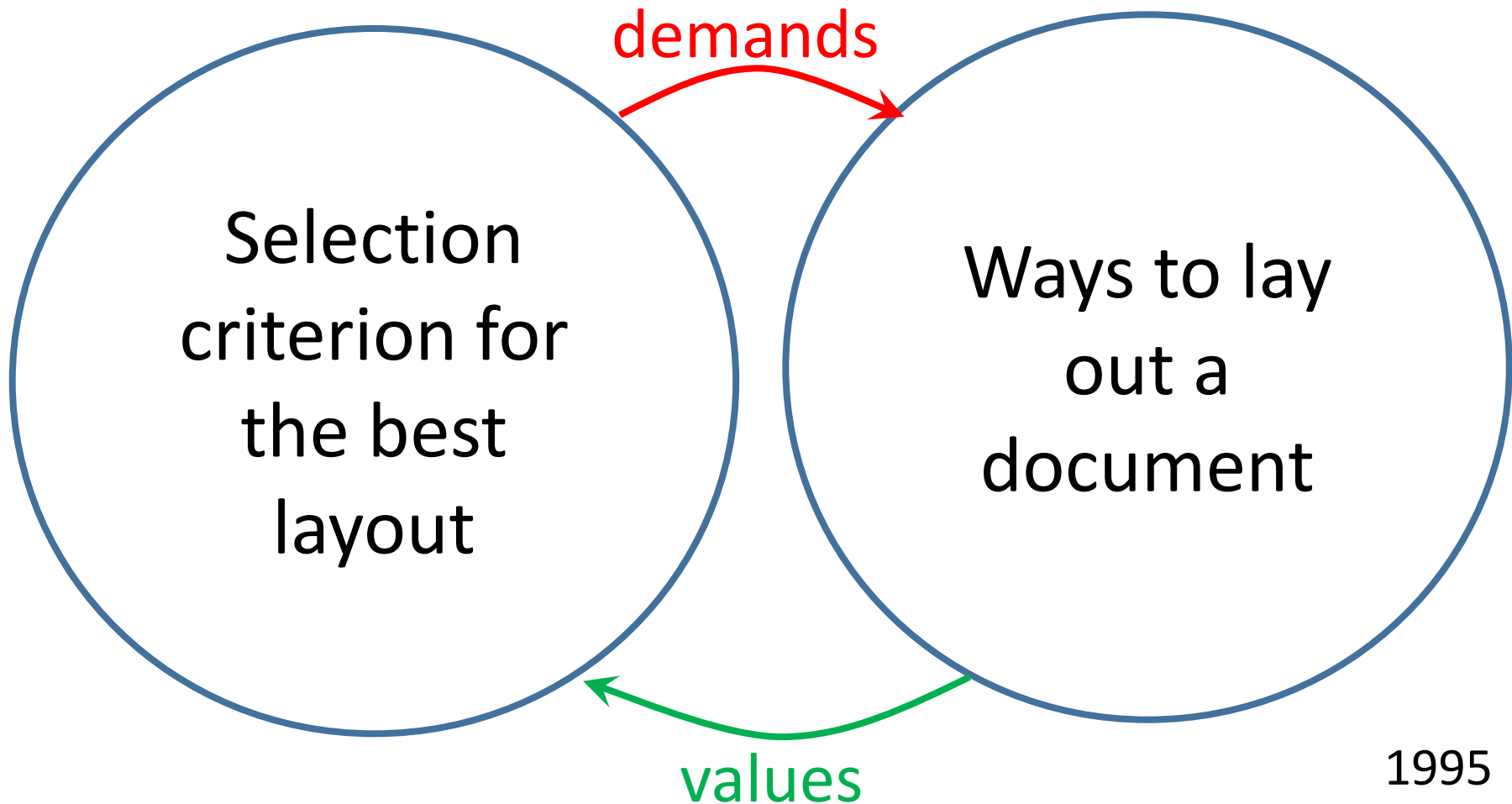
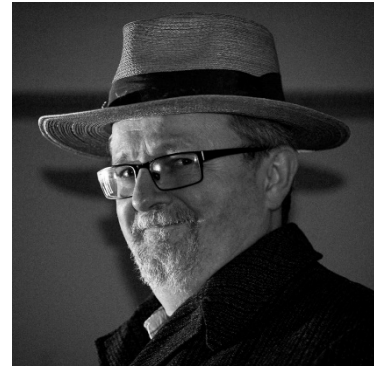
John Hughes  
The University, Glasgow



# The Design of a Pretty-printing Library

John Hughes

Chalmers Tekniska Högskola, Göteborg, Sweden.

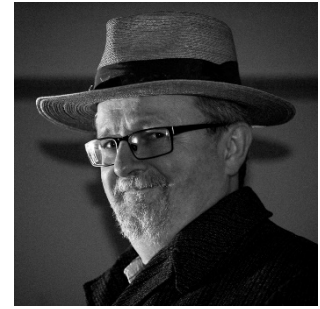


# Improved on by...





# QuickCheck: A Lightweight Tool for Random Testing of Haskell Programs



Koen Claessen  
Chalmers University of Technology  
koen@cs.chalmers.se

John Hughes  
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rjmh@cs.chalmers.se

```
prop_reverse() ->  
  ?FORALL(Xs, list(int()),  
    reverse(reverse(Xs)) == Xs).
```

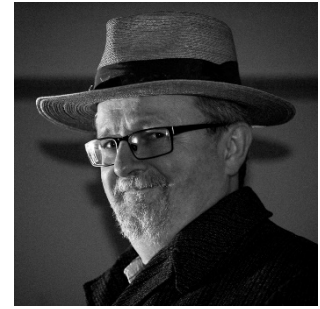
```
3> eqc:quickcheck(qc:prop_reverse()).
```

```
.....  
.....
```

```
OK, passed 100 tests  
true
```



# QuickCheck: A Lightweight Tool for Random Testing of Haskell Programs



Koen Claessen  
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John Hughes  
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rjmh@cs.chalmers.se

```
prop_wrong() ->  
  ?FORALL(Xs, list(int()),  
    reverse(Xs) == Xs).
```

```
4> eqc:quickcheck(qc:prop_wrong()).
```

```
Failed! After 1 tests.
```

```
[-36, -29, 20, 31, -47, -63, 80, -7, 93, -87, -29, 33, 64, 58]
```

```
Shrinking xx.x.x..xx(4 times)
```

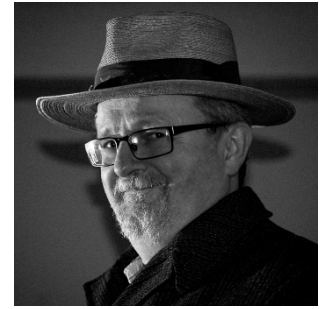
```
[0, 1]
```

```
false
```

*minimal counterexample*

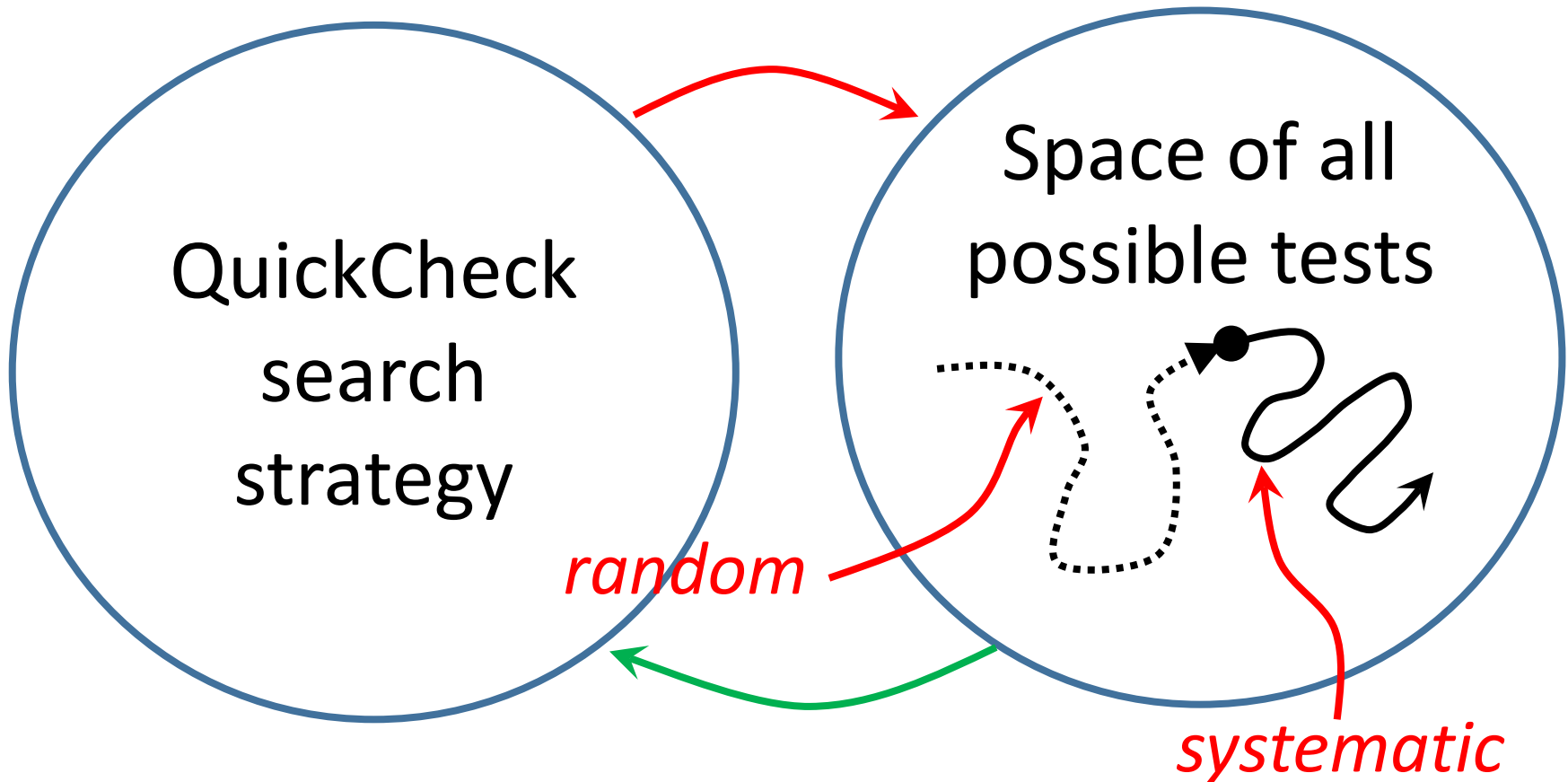


# QuickCheck: A Lightweight Tool for Random Testing of Haskell Programs



Koen Claessen  
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koen@cs.chalmers.se

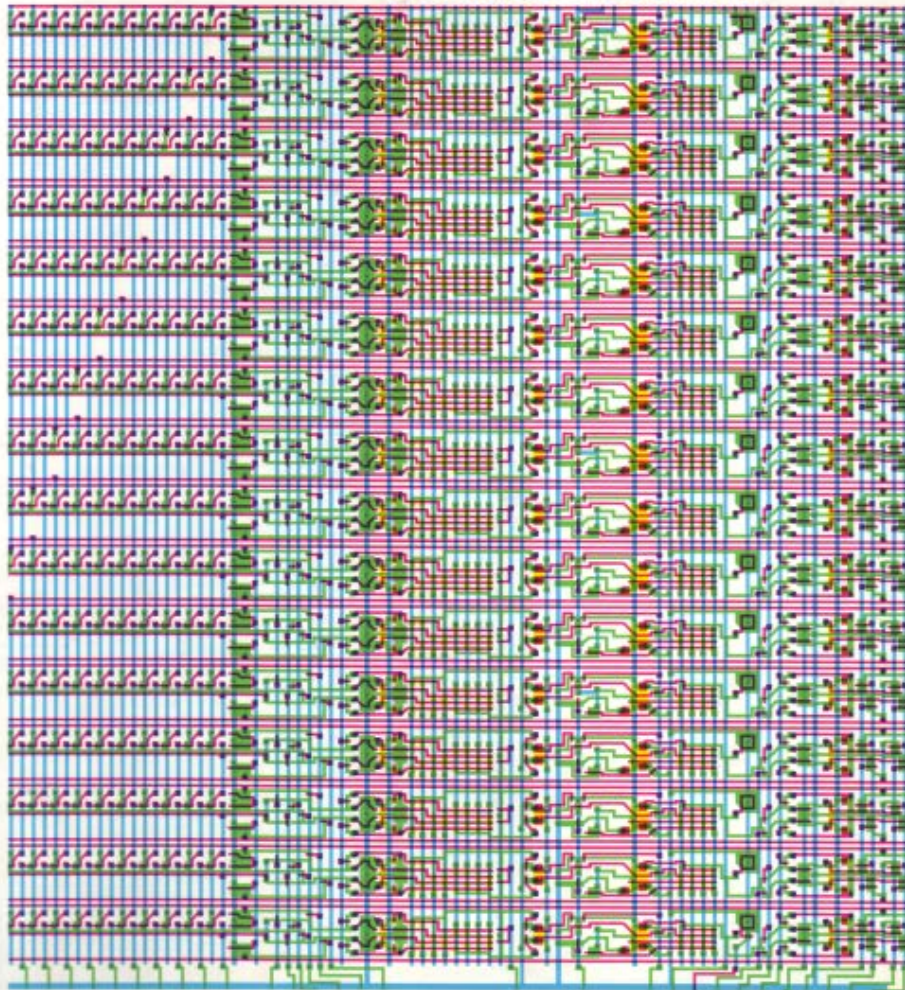
John Hughes  
Chalmers University of Technology  
rjmh@cs.chalmers.se





# INTRODUCTION TO **VLSI** SYSTEMS

CARVER MEAD • LYNN CONWAY

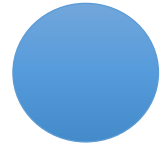




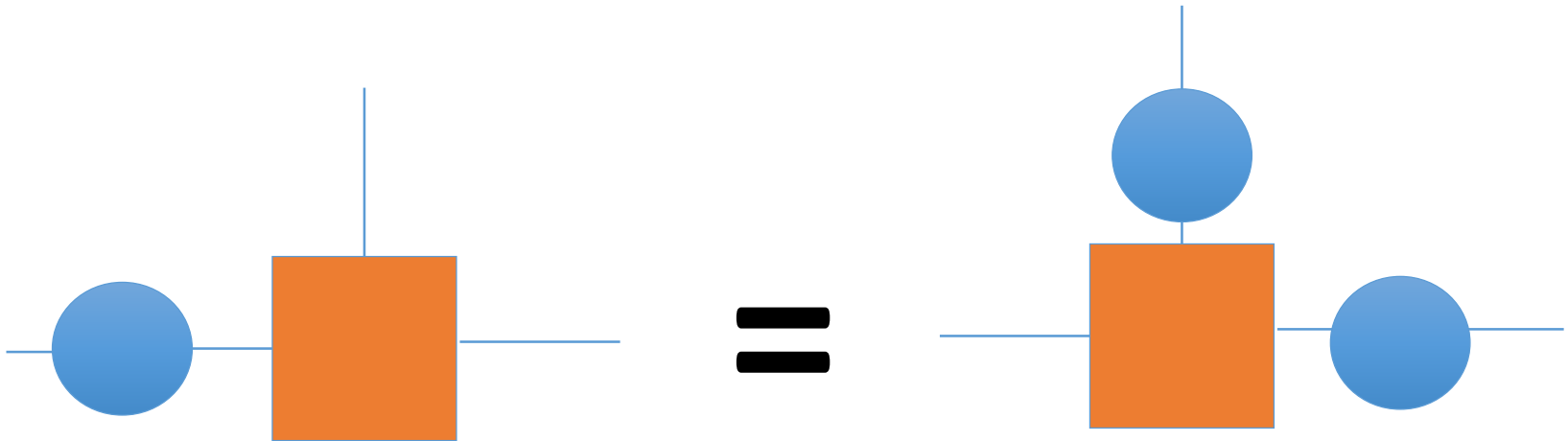
# muFP—Circuits as values

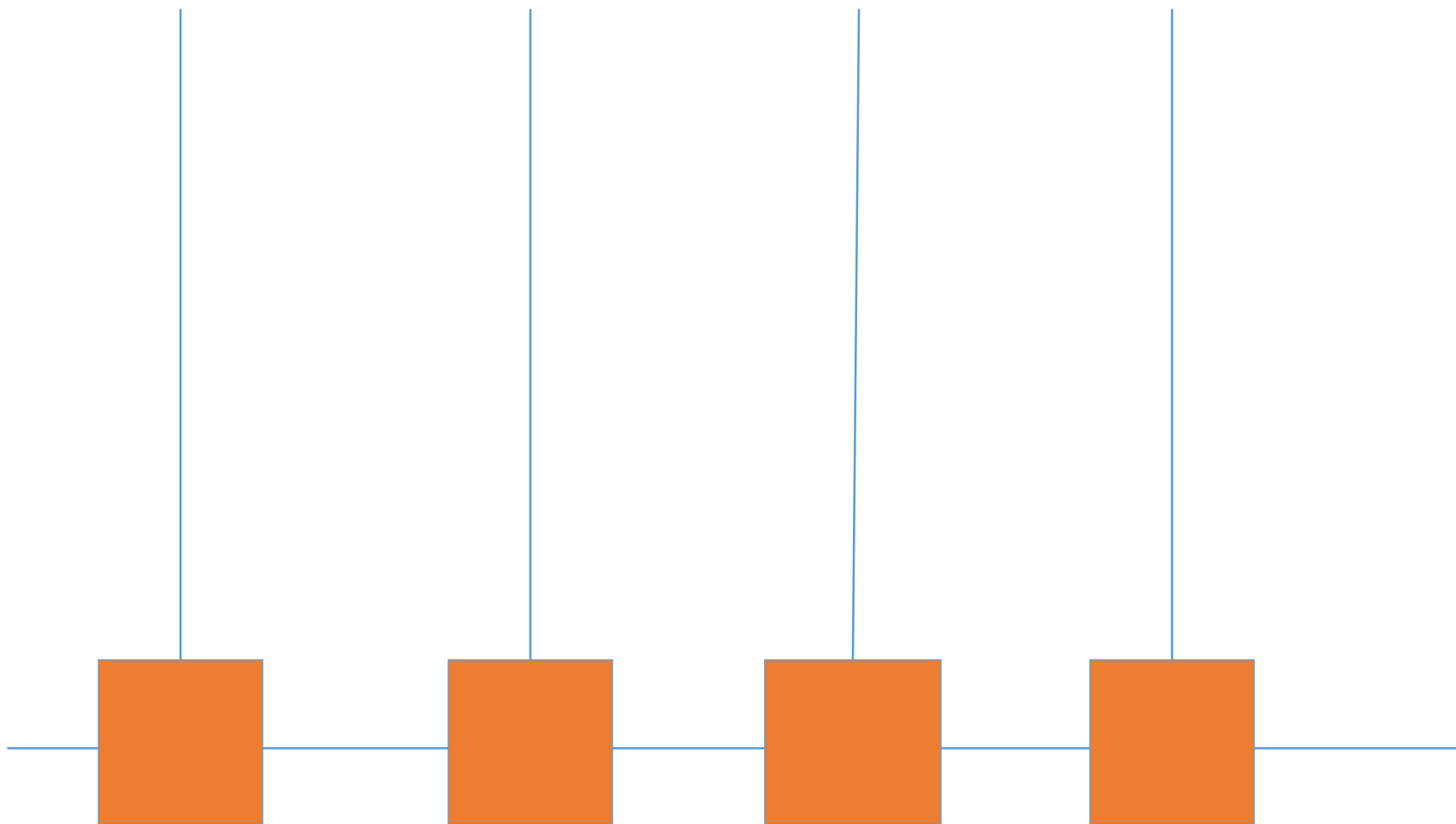


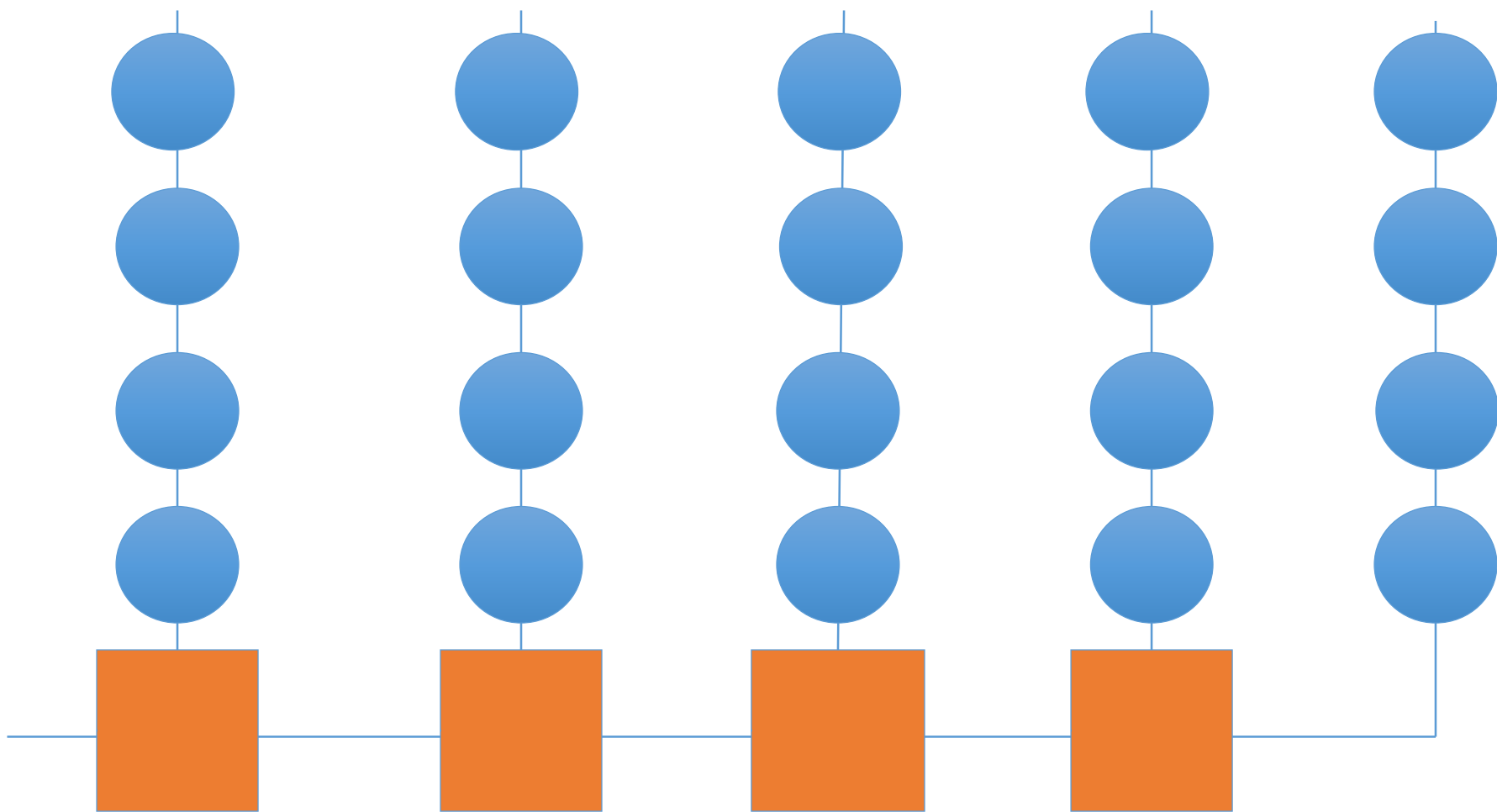
- Backus FP + unit delays
- Inherits many combining forms and laws
- Good for reasoning about alternative designs

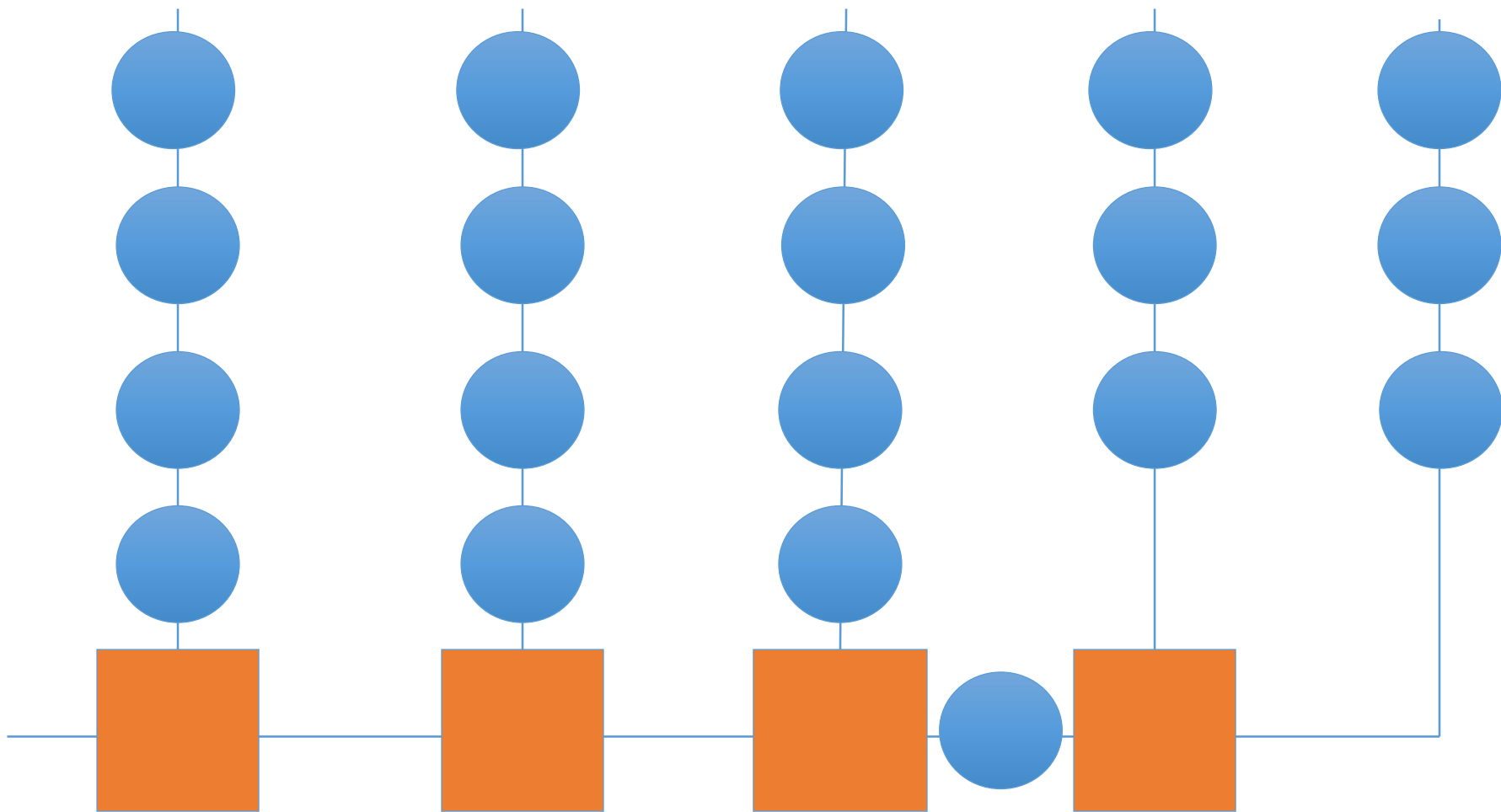


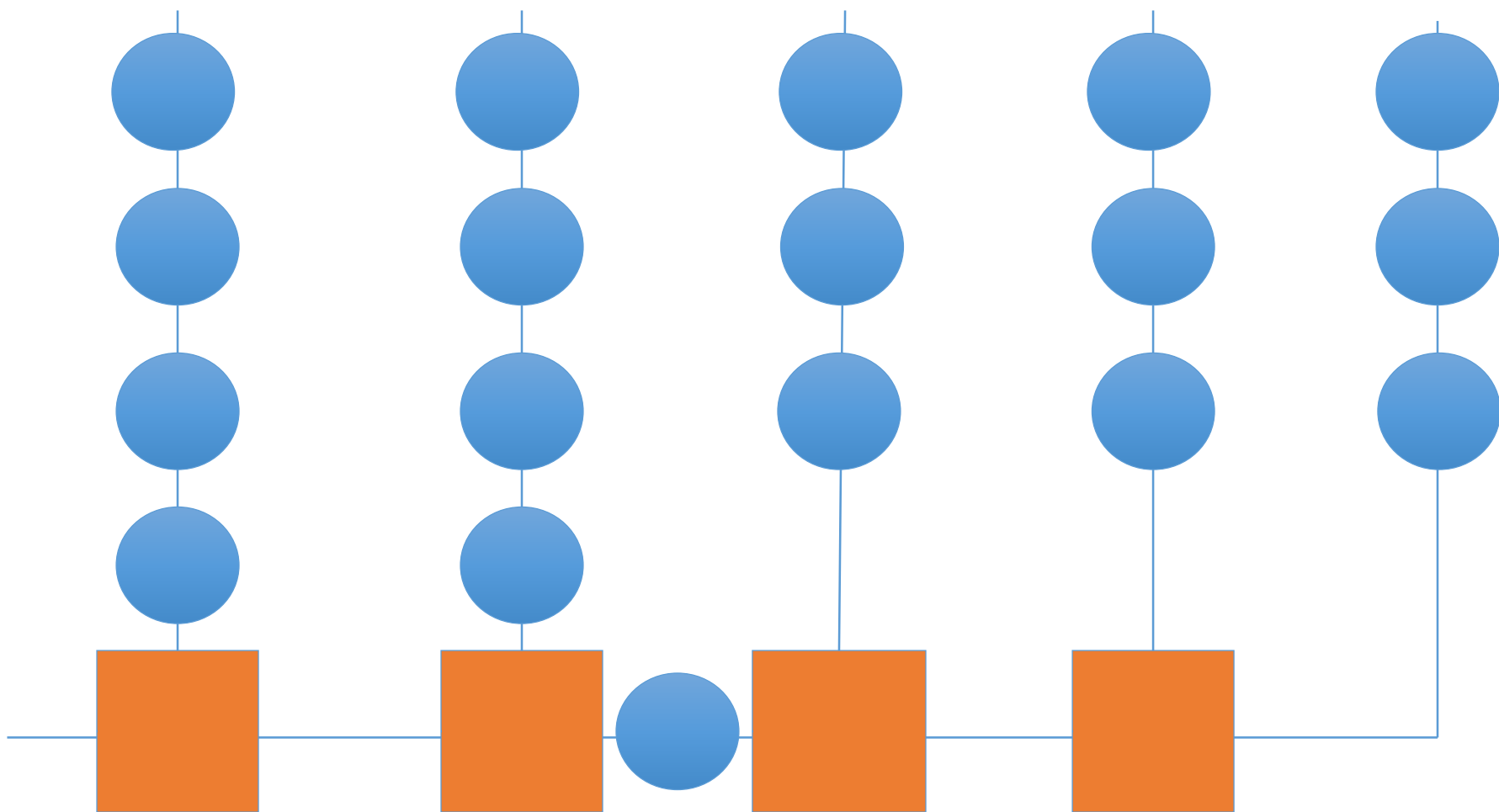
# Example law: “retiming”

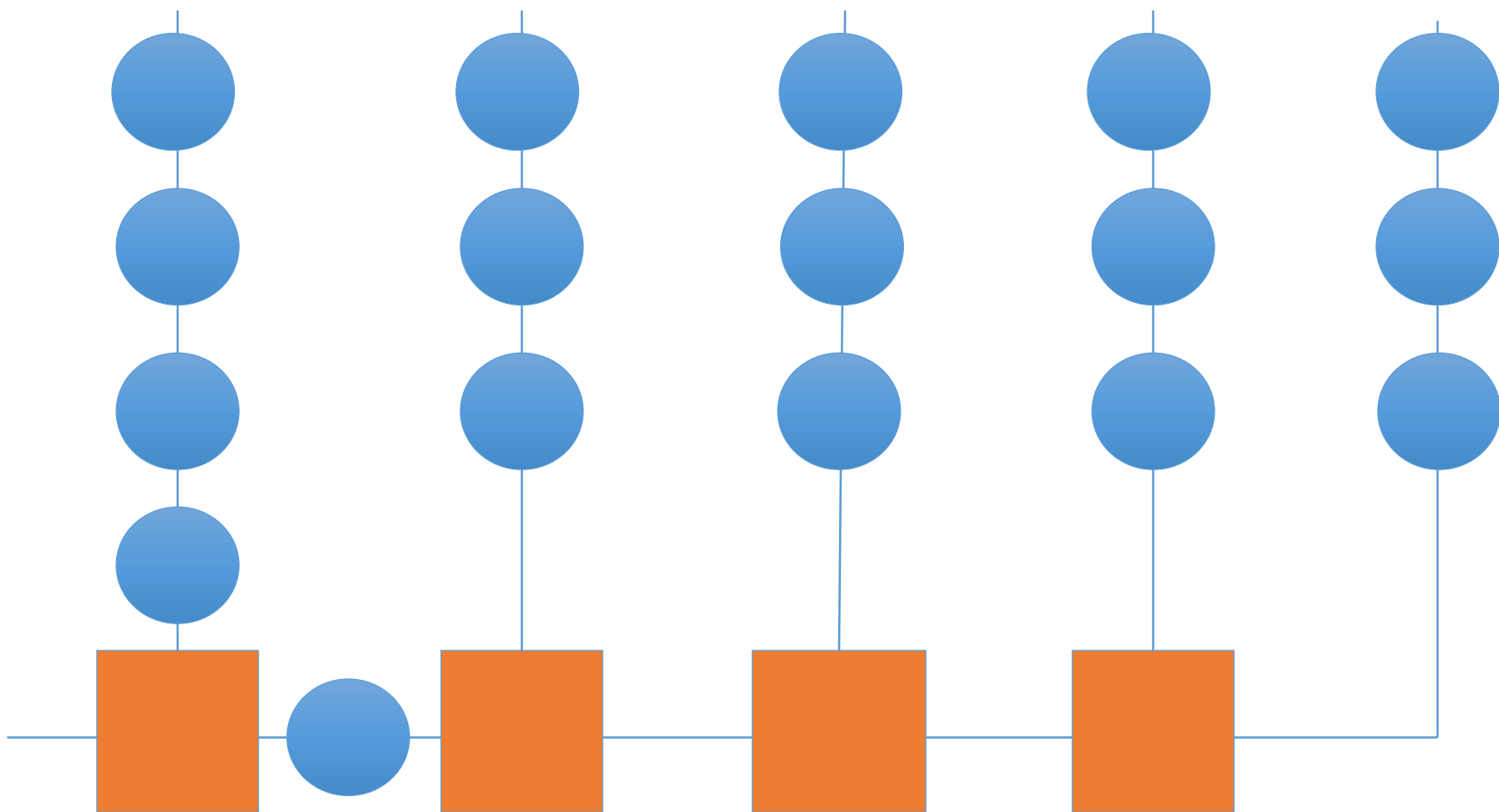


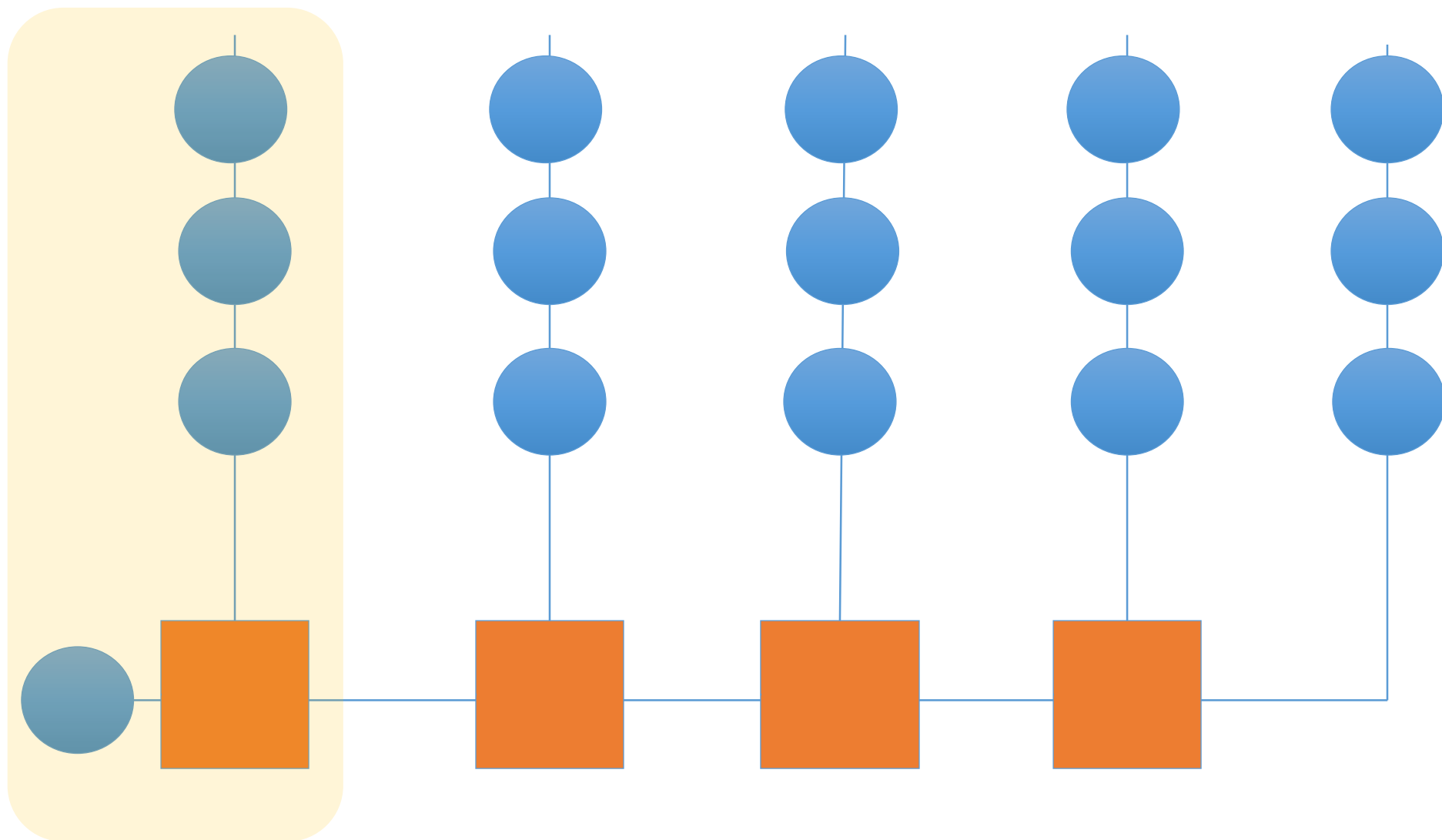




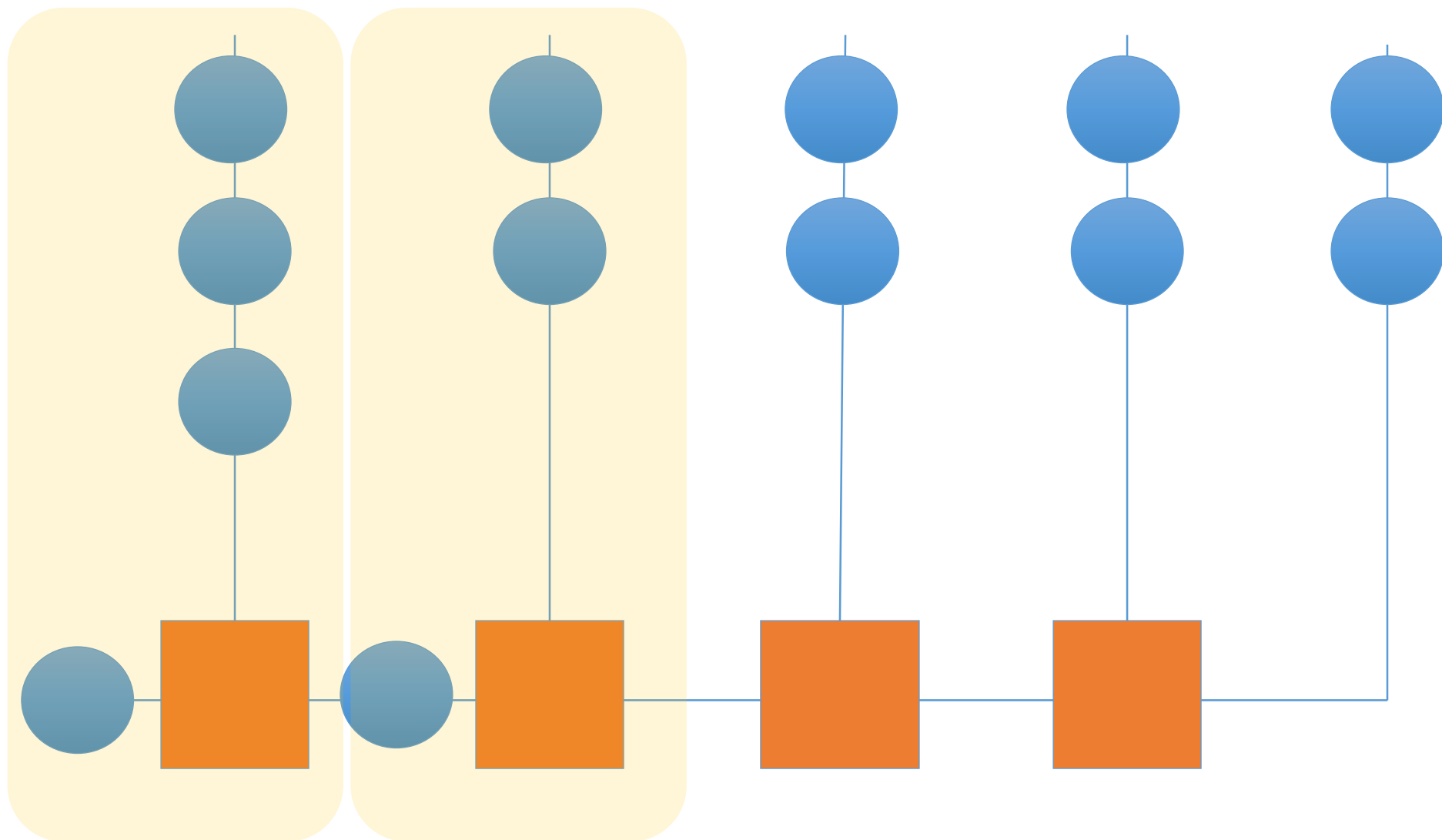


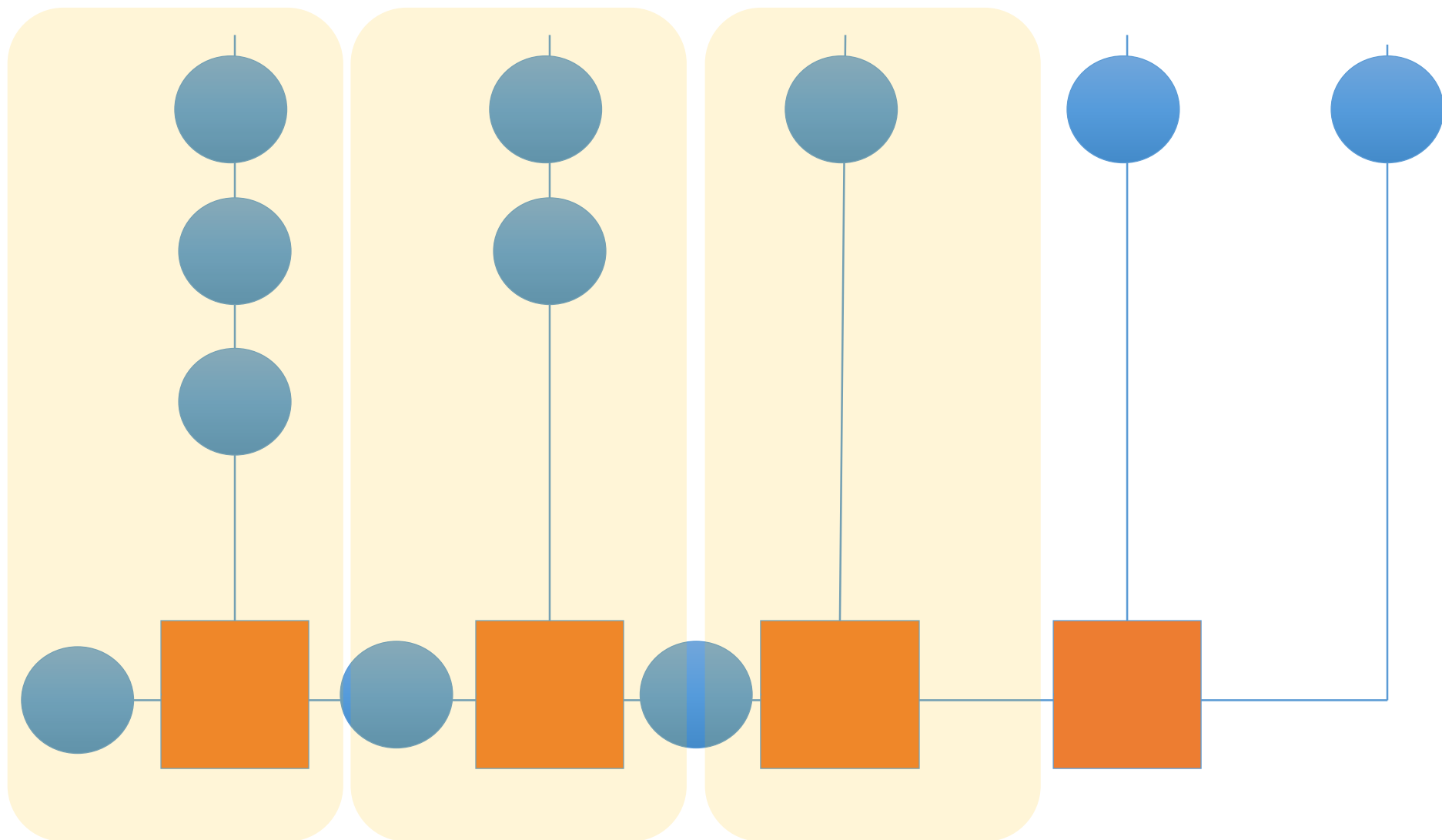


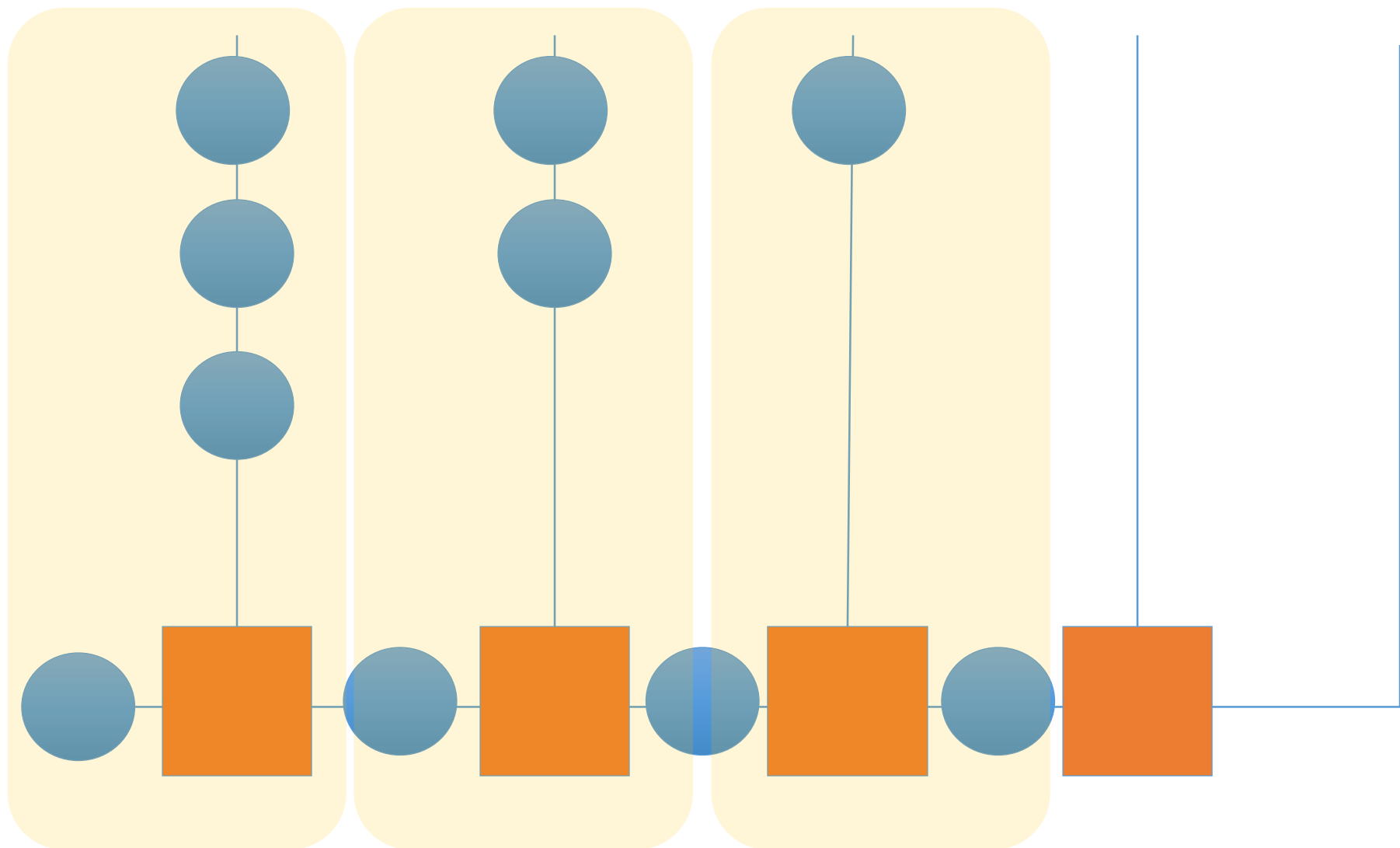












# Users!

## Plessey video motion estimation

“Using muFP, the array processing element was described in just one line of code and the complete array required four lines of muFP description. muFP enabled the effects of adding or moving data latches within the array to be assessed quickly.”

Bhandal et al, An array processor for video picture motion estimation,  
Systolic Array Processors, 1990, Prentice Hall

work with Plessey done by G. Jones and W. Luk

# Lava

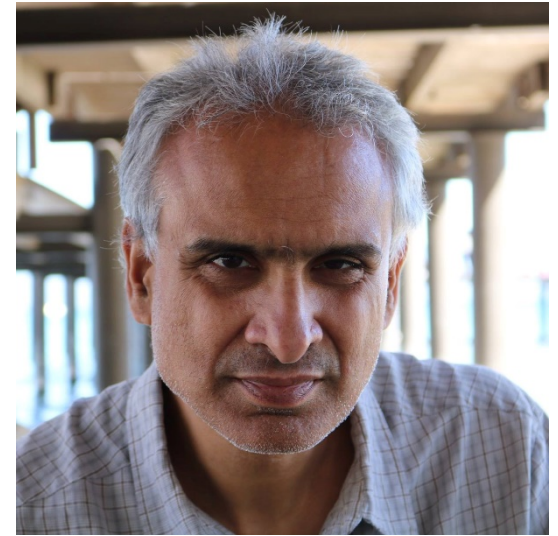
*Semantics*

muFP

+

Functional  
Geometry

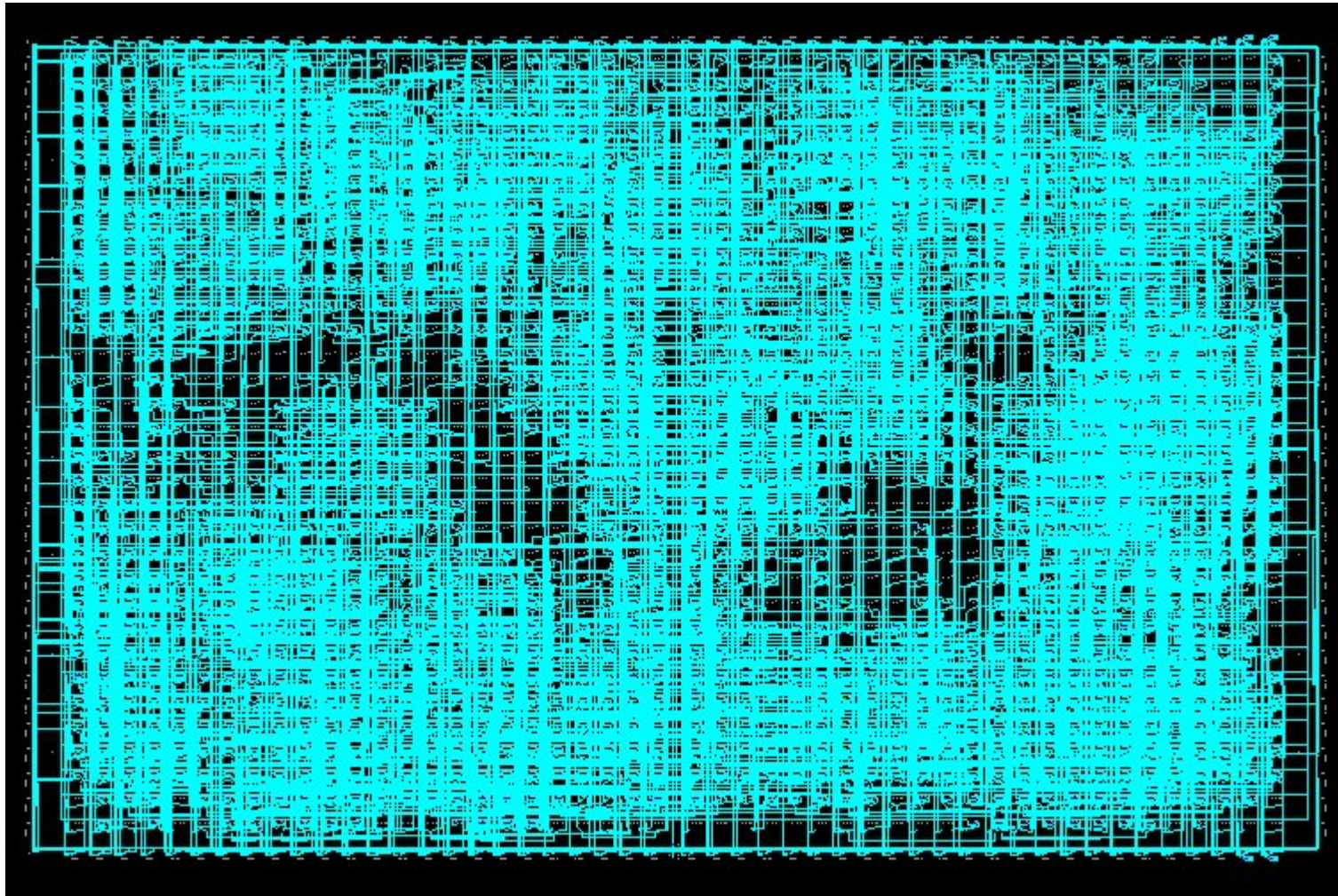
*Placement*



Satnam Singh  
Xilinx

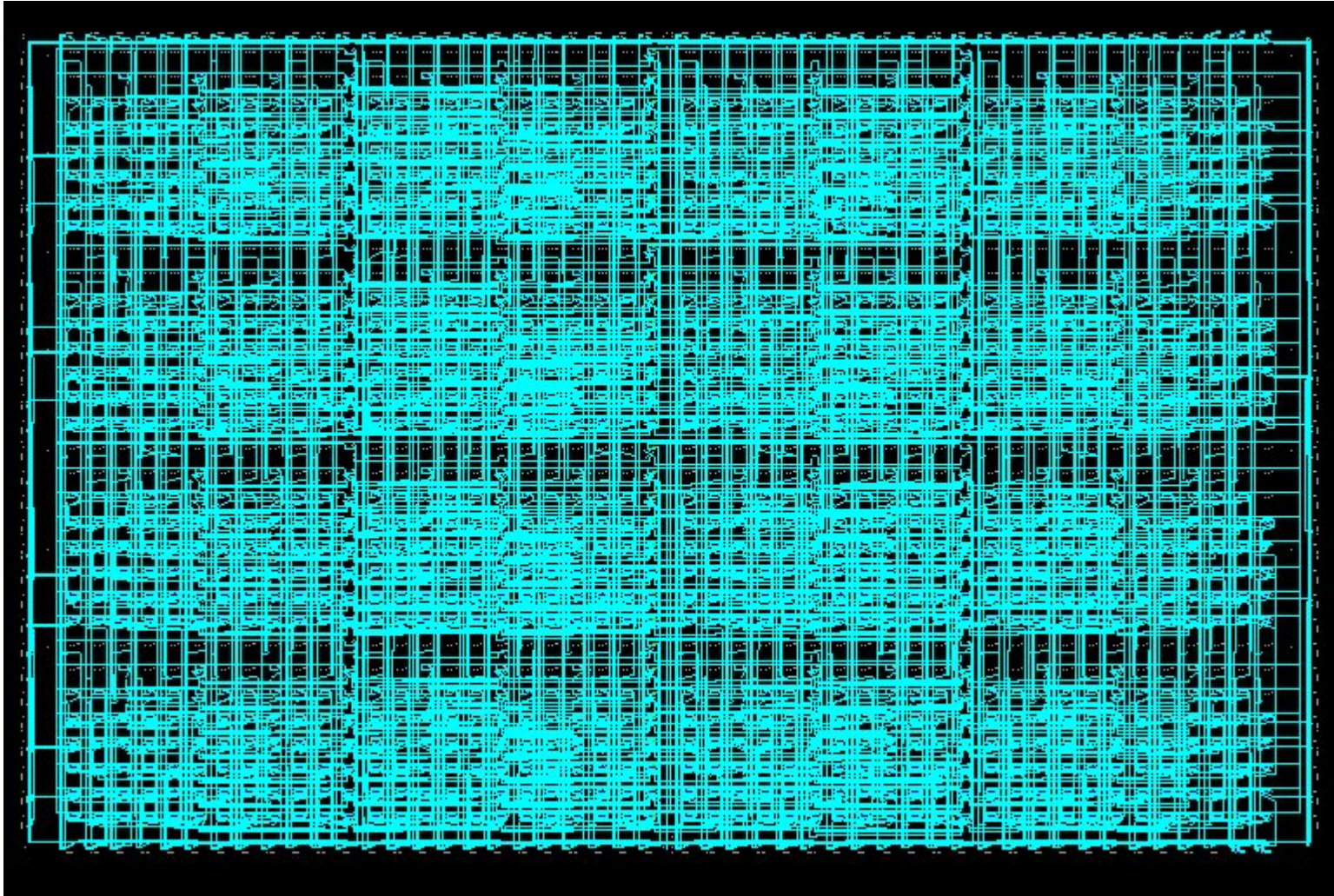
➔ FPGA layouts on Xilinx chips

# Four adder trees—no placement





# Four adder trees—Lava



# Intel

$$4195835.0 - 3145727.0 * (4195835.0 / 3145727.0) = 0$$



# Intel

$$4195835.0 - 3145727.0 * (4195835.0 / 3145727.0) = 0$$

Flawed Pentium

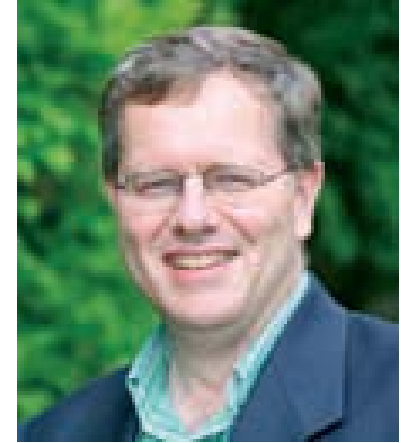
$$4195835.0 - 3145727.0 * (4195835.0 / 3145727.0) = 256$$

# \$475 million



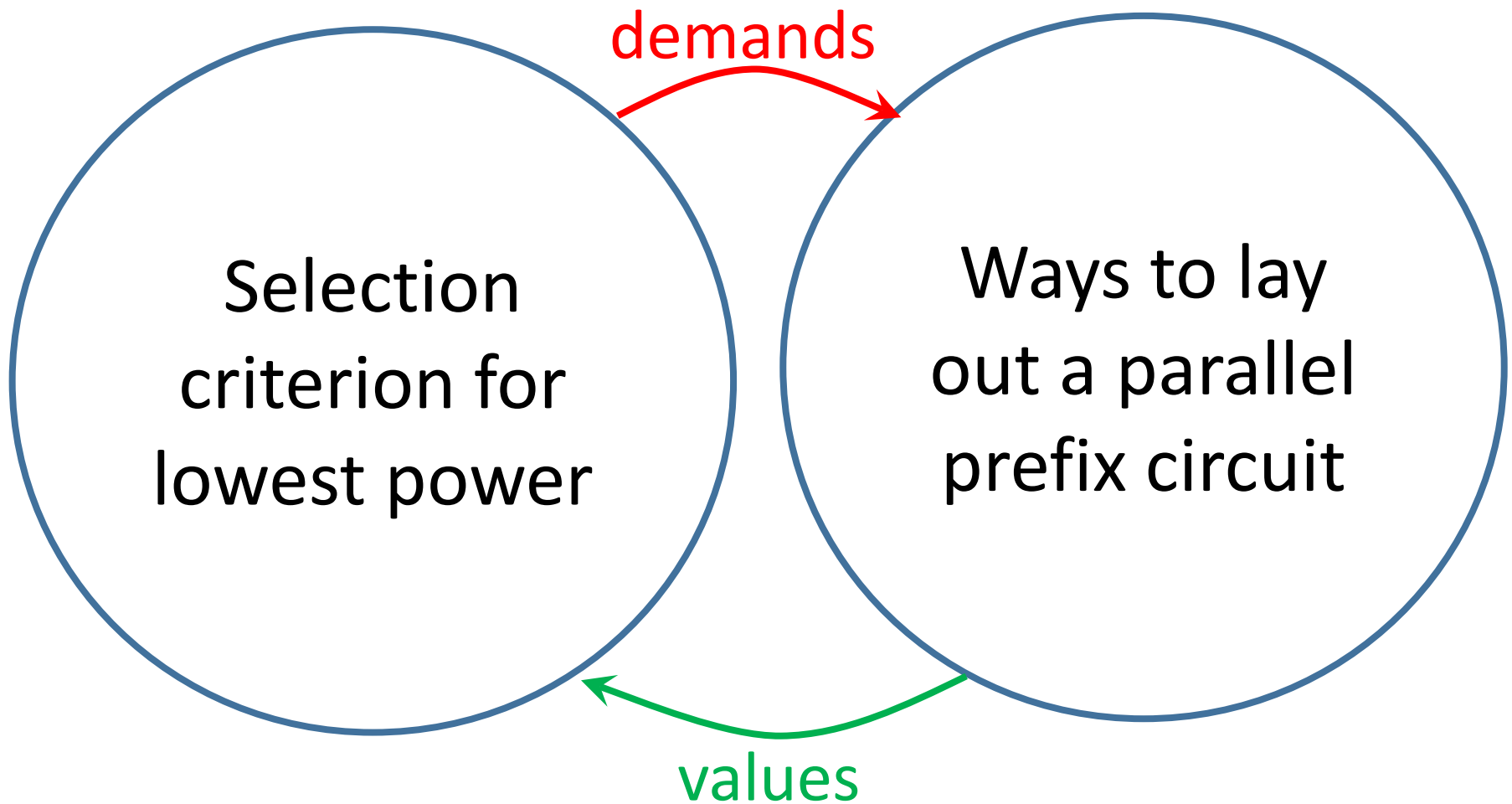
# fl

Lazy functional language,  
1000s users

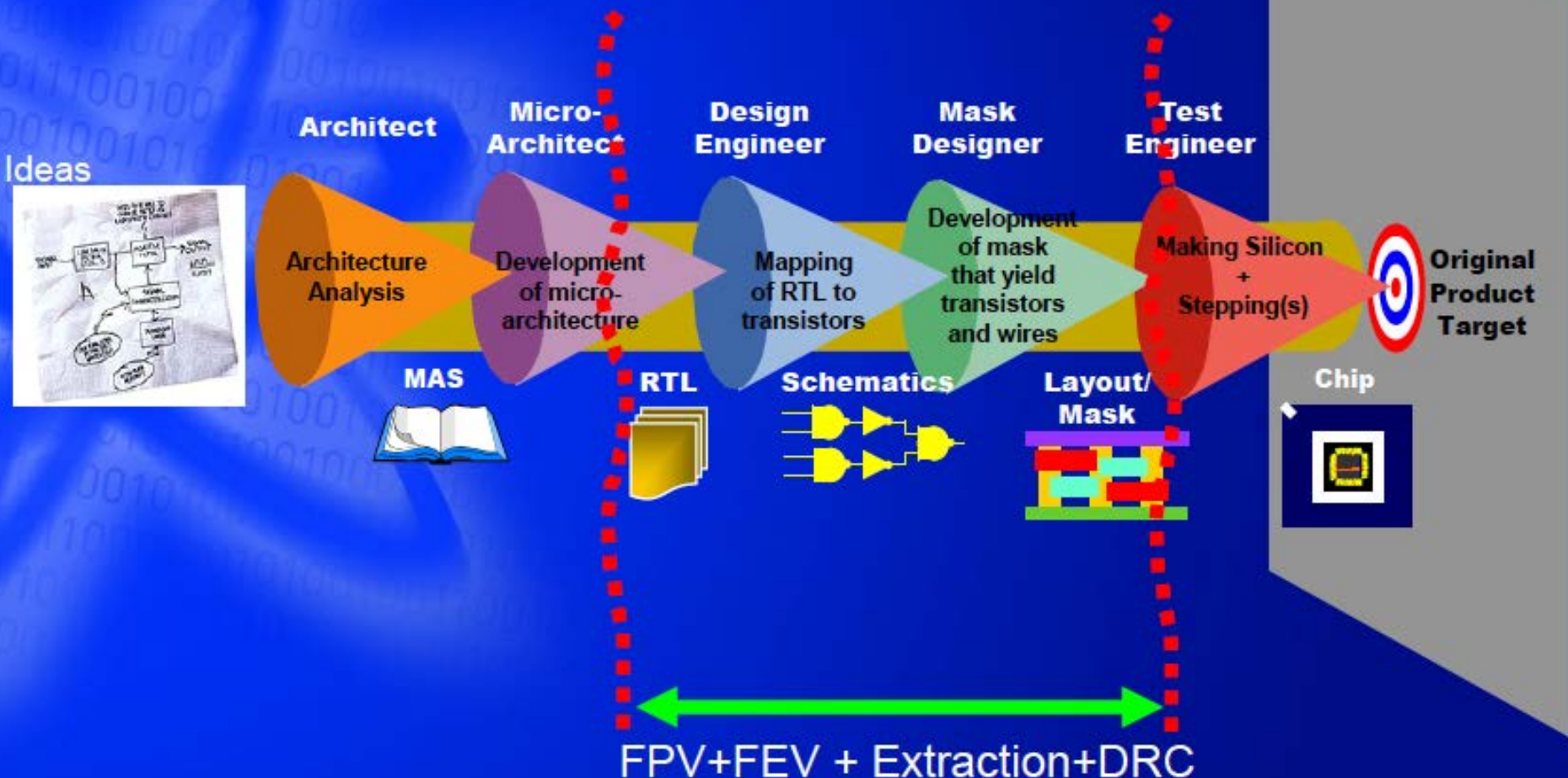


- Design
- High-level specification
- Scripting
- Implementation of formal verification tools and theorem provers
- Object language for theorem proving

Thanks to Carl Seger (formerly of Intel)



# Solid Formal Link with Good Return of the Investment

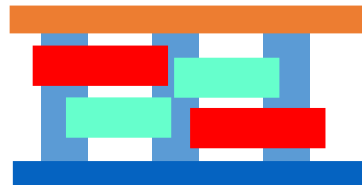


Thanks to Carl Seger (formerly of Intel)

# SPEC



Layout/  
Mask



# Bluespec—FP for hardware



Haskell-like language (architecture) +  
atomic transition rules (H/W modelling)

Frees designers to explore *better algorithms*,  
making major architectural change easy



Types, Functional Programming and  
Atomic Transactions in Hardware Design  
Rishiyur Nikhil LNCS 8000

# Bluecheck

- QuickCheck in Bluespec!
- Generates and shrinks tests *on the FPGA!*

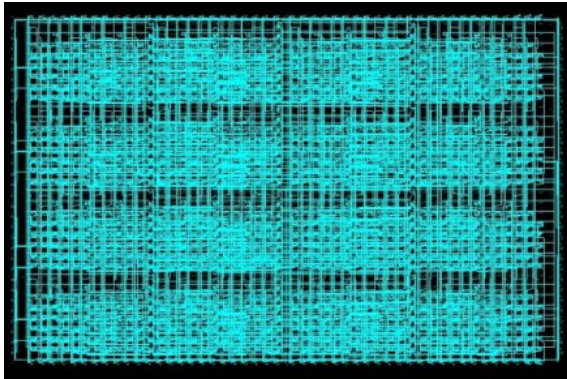
[A Generic Synthesisable Test Bench \(Naylor and Moore, Memocode 2015\)](#)



two  $f\ x = f\ (f\ x)$

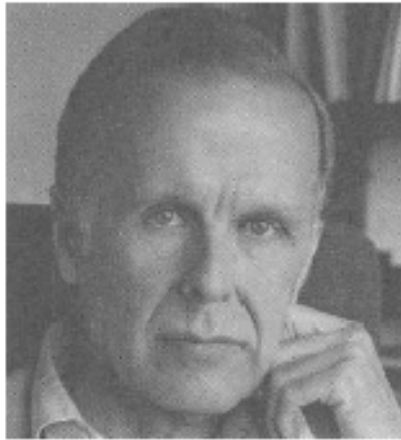
one  $f\ x = f\ x$

zero  $f\ x = x$



**Whole  
values**

**Combining  
forms**



**Simple  
laws**

**Functions as  
representations**