



# Free the Conqueror!

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# Free the Conqueror!

## Refactoring divide-and-conquer functions

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# Content

- Static source code analysis
- Tool-based refactoring
  - **Preserve semantics** while changing the code
  - Avoid *copy-and-paste* and *replace-all* errors
  - Human guided, automatized transformations
  - Faster, easier and more reliable!
  - Bounded expressiveness

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# Divide-and-Conquer

Split problem into smaller ones and recurse!

- Sorting (Quicksort, Mergesort, Bucketsort...)
- Mini-max search
- Karatsuba-multiplication
- ...

Many possible applications in HPC!

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# Divide-and-Conquer structure

```
solve(Problem) =  
  if is_base_case(Problem)  
  then solve_base_case(Problem)  
  else SubProblems = divide(Problem)  
       Solutions = map(solve, SubProblems)  
       combine(Solutions)  
  end
```

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# Divide-and-Conquer HOF

```
solve = dc( is_base_case, solve_base_case,  
           divide,           combine )
```

```
dc(IsBase, Base, Divide, Combine) =  
  let Solve(Problem) =  
    if IsBase(Problem)  
    then Base(Problem)  
    else SubProblems = Divide(Problem)  
         Sols = map(Solve, SubProblems)  
         Combine(Sols)  
    end  
  in Solve
```

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# Motivation

- Highly heterogeneous mega-core computers
- Pattern-based parallelism

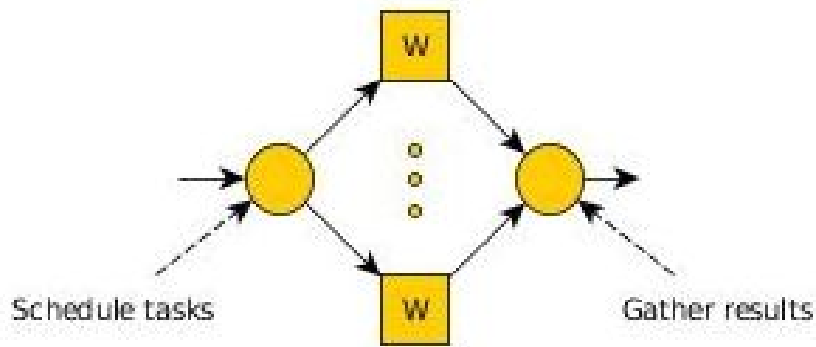


## Parallel Patterns for Adaptive Heterogeneous Multicore Systems

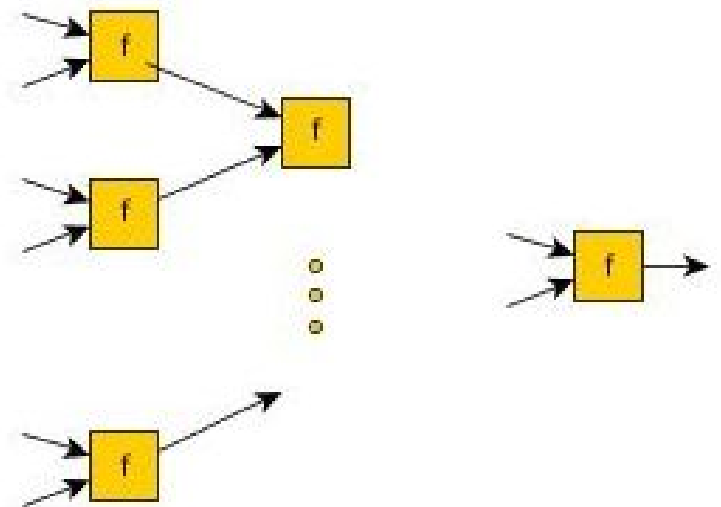


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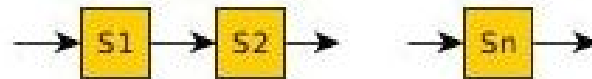
### Farm



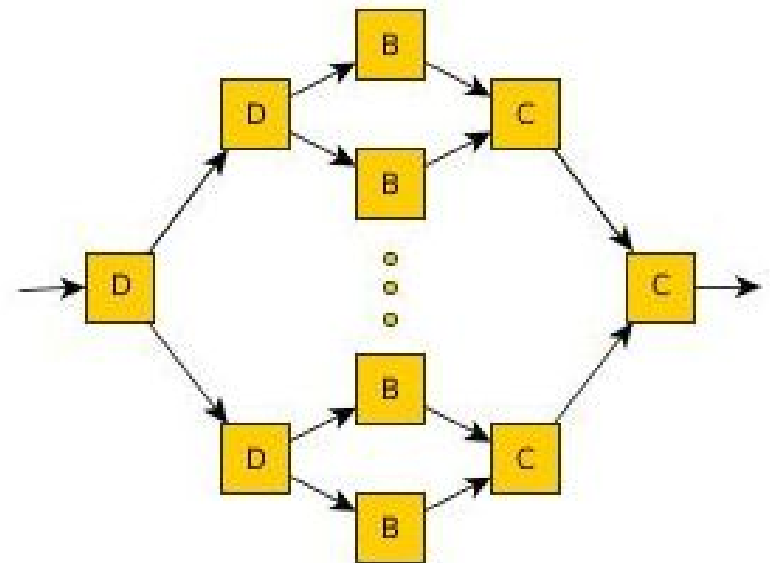
### Reduce



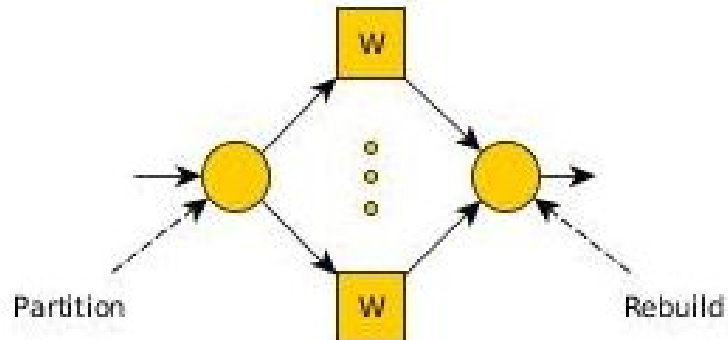
### Pipeline



### Divide&Conquer



### Map



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# Mergesort



```
ms ( [] ) -> [];
```

```
ms ( [H] ) -> [H];
```

```
ms ( [H|T]=L ) ->
```

```
  {LL,LR} = lists:split(length(L) div 2, L),  
  merge( ms(LL), ms(LR) ).
```

```
merge( L1, L2 ) -> ...
```

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# Mergesort with d&c pattern

```
ms(List) ->
  (skel_hlp:dc(
    fun(L) -> length(L) < 2 end, % base case?
    fun(L) -> L end,             % base case
    fun(L) ->                    % divide
      {LL,LR}=lists:split(length(L) div 2, L),
      [LL,LR] end,
    fun([L1,L2]) ->             % combine
      merge(L1,L2) end
  ))(List).
```

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# Divide-and-Conquer (seq)

```
dc(IsBase,Base,Divide,Combine) ->
```

```
  Knot =
```

```
    fun(Self,Problem) ->
```

```
      case IsBase(Problem) of
```

```
        true ->
```

```
          Base(Problem);
```

```
        false ->
```

```
          PS = Divide(Problem),
```

```
          SS = [Self(Self,P) || P <- PS]
```

```
          Combine(SS)
```

```
      end
```

```
    end,
```

```
  fun(Problem) -> Knot(Knot,Problem) end.
```

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# Divide-and-Conquer (par)

```
dc(IsBase,Base,Divide,Combine)
```

```
dc(IsBase,Base,Divide,Combine,NrProc)  
% at most NrProc processes
```

```
dc(IsSeq,IsBase,Base,Divide,Combine)  
% switch to sequential at IsSeq
```

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# We provide tooling to...

- **F**ind divide-and-conquer pattern candidates
- **A**ssist in making parallelization decisions
- **R**efactor candidates to the pattern

PaRTE

ParaPhrase Refactoring Tool for Erlang

(Powered by RefactorErl)

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# Standard Refactorings

- Rename/Move definitions
- Introduce/Eliminate function/variable
- Generalize function
- Reorder/Tuple function parameters

plus many Erlang-specific transformations,  
altogether 24 refactorings

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# Intro Divide-and-Conquer

- Find functions which are d&c (candidates)
- Transform them step-by-step into *canonical* form

```
ms( L ) ->
```

```
  case isBase(L) of
```

```
    true  -> base(L);
```

```
    false ->
```

```
      Problems = divide(L),
```

```
      Solutions = lists:map(fun ms/1, Problems),
```

```
      combine(Solutions)
```

```
  end.
```

- Replace canonical form with call to skeleton

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# D&C candidate

- Our definition:
  - a function activates itself multiple times on the same execution path, with parameters not depending on recursive calls
- Many syntactical possibilities!
  - explicit recursive calls
  - lists:map or list comprehensions
  - mutual recursion

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# Karatsuba

karatsuba( Num1 , Num2 ) ->

...

Z0 = karatsuba( Low1 , Low2 ),

Z1 = karatsuba( add(Low1,High1),  
add(Low2,High2) ),

Z2 = karatsuba( High1 , High2 ),

...

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# Minimax

```
mm_max( Node, Depth ) ->  
  case Depth == 0 orelse terminal(Node)  
    true   -> value ( Node ) ;  
    false -> lists:max([ mm_min(C, Depth-1)  
                       || C <- children(Node)  
                       ])  
  
  end.
```

```
mm_min( Node, Depth ) -> ... mm_max ...
```

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# Novel refactorings

- Function clauses to case expression
- Group case branches
- Bindings to list
- Introduce lists:map/2
- Merge function definitions
- Move in/out expression to/from case
- Case on list comprehension
- Eliminate single branch in case expression

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# Function clauses to case expr.

```
ms( [] ) -> [];
```

```
ms( [H] ) -> [H];
```

```
ms( [H|T]=L ) ->
```

```
  {LL,LR} = lists:split(length(L) div 2, L),  
  merge( ms(LL), ms(LR) ).
```

```
merge( ... , ... ) -> ...
```

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# Function clauses to case expr.

```
ms( L ) ->  
  case L of  
    []      -> [];  
    [H]     -> [H];  
    [H|T]   -> {LL,LR} = lists:split(  
                                     length(L) div 2, L),  
               merge( ms(LL), ms(LR) )  
  end.
```



# Group case branches

```
ms( L ) ->  
  case L of  
    []      -> [];  
    [H]     -> [H];  
    [H|T]   -> {LL,LR} = lists:split(  
                                   length(L) div 2, L),  
                                   merge( ms(LL), ms(LR) )  
  end.
```

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```
ms( L ) ->
  IsBase = case L of
    []      -> true;
    [H]     -> true;
    [H|T]   -> false
  end,
  case IsBase of
    true   -> case L of
      []    -> [];
      [H]   -> [H]
    end;
    false  -> case L of
      [H|T] -> {LL,LR} = lists:split(
                    length(L) div 2, L),
                    merge( ms(LL), ms(LR) )
    end
  end.
```

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# Introduce function

```
ms( L ) ->
  IsBase = case L of
    []      -> true;
    [H]     -> true;
    [H|T]   -> false
  end,
  case IsBase of
    true -> case L of
      []      -> [];
      [H]     -> [H]
    end;
    false -> case L of
      [H|T] -> {LL,LR} = lists:split(
                          length(L) div 2, L),
                          merge( ms(LL), ms(LR) )
    end
  end.
```

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# Introduce function

```
isBase( L ) ->
  case L of
    [] -> true;
    [H] -> true;
    [H|T] -> false
  end.

ms( L ) ->
  IsBase = isBase(L),
  case IsBase of
    true -> case L of
      [] -> [];
      [H] -> [H]
    end;
    false -> case L of
      [H|T] -> {LL,LR} = lists:split(
        length(L) div 2, L),
        merge( ms(LL), ms(LR) )
    end
  end.
end.
```

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# Eliminate variable

```
isBase( L ) -> case L of
    []      -> true;
    [H]     -> true;
    [H|T]   -> false
end.

ms( L ) ->
  IsBase = isBase(L),
  case IsBase of
    true  -> case L of
        []      -> [];
        [H]     -> [H]
      end;
    false -> case L of
        [H|T] -> {LL,LR} = lists:split(
            length(L) div 2, L),
            merge( ms(LL), ms(LR) )
      end
  end.
end.
```

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# Eliminate variable

```
isBase( L ) -> case L of
    []      -> true;
    [H]     -> true;
    [H|T]   -> false
end.

ms( L ) ->
  case isBase(L) of
    true  -> case L of
        []      -> [];
        [H]     -> [H]
        end;
    false -> case L of
        [H|T] -> {LL,LR} = lists:split(
            length(L) div 2, L),
            merge( ms(LL), ms(LR) )
        end
    end.
end.
```

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# Canonical form

```
ms( L ) ->  
  case isBase(L) of  
    true  -> solve(L);  
    false -> Problems = divide(L),  
           Solutions = lists:map(fun ms/1, Problems),  
           combine(Solutions)  
  
  end.
```

```
combine( ListOfLists ) -> ...  
solve ( L ) -> ...  
isBase( L ) -> ...  
divide( L ) -> ...
```

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# Divide-and-Conquer pattern

```
ms( L ) ->  
  (skel_hlp:dc( fun isBase/1,  
               fun solve/1,  
               fun divide/1,  
               fun combine/1 ))(L).
```

```
combine( ListOfLists ) -> ...  
solve ( L ) -> ...  
isBase( L ) -> ...  
divide( L ) -> ...
```

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# Merge function definitions

```
mm_max( {Node, Depth} ) ->
```

```
  case Depth == 0 or else terminal(Node)
```

```
    true  -> value ( Node ) ;
```

```
    false -> lists:max([ mm_min({C, Depth-1})  
                        || C <- children(Node)  
                        ])
```

```
end.
```

```
mm_min( {Node, Depth} ) ->
```

```
    ... mm_max ...
```

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# Merge function definitions

```
mm( {Node, Depth}, max ) ->  
  case Depth == 0 orelse terminal(Node)  
    true  -> value ( Node ) ;  
    false -> lists:max([ mm({C, Depth-1}, min)  
                        || C <- children(Node)  
                        ])  
  
  end;
```

```
mm( {Node, Depth}, min ) ->  
  ... mm( ... , max ) ...
```

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# Conclusions

- Pattern-based parallelism
- Pattern discovery and refactoring tool
- Step-by-step refactoring of D&C functions
  - Programmer guided, performed with tool
  - Preserving semantics (proof?)
  - Small but meaningful transformations
    - Generic or d&c-specific
  - Compound transformation?
  - Manual refactoring can come between

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# Mergesort with processes

```
ms( [H|T]=L ) ->  
  {LL,LR} = lists:split(length(L) div 2, L),  
  Parent = self(),  
  spawn( fun() -> Parent ! ms(LL) end ),  
  spawn( fun() -> Parent ! ms(LR) end ),  
  receive L1 -> ok end,  
  receive L2 -> ok end,  
  merge( L1, L2 ).
```

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# Refactoring Mergesort

```
ms( [] ) -> [];  
ms( [H] ) -> [H];  
ms( [H|T]=L ) ->  
    {LL,LR} = lists:split(length(L) div 2, L),  
    merge( ms(LL), ms(LR) ).  
  
merge( ... , ... ) -> ...
```

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# Function clauses to case expr.

```
ms( [] ) -> [];
```

```
ms( [H] ) -> [H];
```

```
ms( [H|T]=L ) ->
```

```
  {LL,LR} = lists:split(length(L) div 2, L),
```

```
  merge( ms(LL), ms(LR) ).
```

```
merge( ... , ... ) -> ...
```

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# Function clauses to case expr.

```
ms( L ) ->  
  case L of  
    []      -> [];  
    [H]     -> [H];  
    [H|T]   -> {LL,LR} = lists:split(  
                                     length(L) div 2, L),  
                                     merge( ms(LL), ms(LR) )  
  end.
```



# Group case branches

```
ms( L ) ->  
  case L of  
    []      -> [];  
    [H]     -> [H];  
    [H|T]   -> {LL,LR} = lists:split(  
                                     length(L) div 2, L),  
                                     merge( ms(LL), ms(LR) )  
  end.
```

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```
ms( L ) ->
  IsBase = case L of
    []      -> true;
    [H]     -> true;
    [H|T]   -> false
  end,
  case IsBase of
    true   -> case L of
      []    -> [];
      [H]   -> [H]
    end;
    false -> case L of
      [H|T] -> {LL,LR} = lists:split(
                    length(L) div 2, L),
                    merge( ms(LL), ms(LR) )
    end
  end.
```

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# Introduce function

```
ms( L ) ->
  IsBase = case L of
    []      -> true;
    [H]     -> true;
    [H|T]   -> false
  end,
  case IsBase of
    true   -> case L of
      []    -> [];
      [H]   -> [H]
    end;
    false -> case L of
      [H|T] -> {LL,LR} = lists:split(
                    length(L) div 2, L),
                    merge( ms(LL), ms(LR) )
    end
  end.
```

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# Introduce function

```
isBase( L ) -> case L of
    []      -> true;
    [H]     -> true;
    [H|T]   -> false
end.

ms( L ) ->
  IsBase = isBase(L),
  case IsBase of
    true  -> case L of
        []      -> [];
        [H]     -> [H]
      end;
    false -> case L of
        [H|T] -> {LL,LR} = lists:split(
            length(L) div 2, L),
            merge( ms(LL), ms(LR) )
      end
  end.
end.
```

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# Eliminate variable

```
ms( L ) ->
  IsBase = isBase(L),
  case IsBase of
    true  -> case L of
      []   -> [];
      [H]  -> [H]
    end;
  false -> case L of
    [H|T] -> {LL,LR} = lists:split(
      length(L) div 2, L),
      merge( ms(LL), ms(LR) )
    end
  end.
```

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# Eliminate variable

```
ms( L ) ->
  case isBase(L) of
    true  -> case L of
      []   -> [];
      [H]  -> [H]
    end;
  false -> case L of
    [H|T] -> {LL,LR} = lists:split(
      length(L) div 2, L),
      merge( ms(LL), ms(LR) )
    end
  end.
```

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# Introduce function

```
ms( L ) ->
  case isBase(L) of
    true  -> case L of
      []   -> [];
      [H]  -> [H]
    end;
  false -> case L of
    [H|T] -> {LL,LR} = lists:split(
      length(L) div 2, L),
      merge( ms(LL), ms(LR) )
    end
  end.
```

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# Introduce function

```
ms( L ) ->  
  case isBase(L) of  
    true  -> base(L);  
    false -> case L of  
      [H|T] -> {LL,LR} = lists:split(  
                    length(L) div 2, L),  
                    merge( ms(LL), ms(LR) )  
    end  
  end.
```

```
base( L ) -> case L of  
  []      -> [];  
  [H]     -> [H]  
end.
```

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# Eliminate case expression

```
ms( L ) ->  
  case isBase(L) of  
    true  -> base(L);  
    false -> case L of  
      [H|T] -> {LL,LR} = lists:split(  
                    length(L) div 2, L),  
                    merge( ms(LL), ms(LR) )  
    end  
  end.
```



# Eliminate case expression

```
ms( L ) ->  
  case isBase(L) of  
    true  -> base(L);  
    false -> [H|T] = L,  
             {LL,LR} = lists:split(length(L) div 2, L),  
             merge( ms(LL), ms(LR) )  
  end.
```

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# Introduce function

```
ms( L ) ->  
  case isBase(L) of  
    true  -> base(L);  
    false -> [H|T] = L,  
             {LL,LR} = lists:split(length(L) div 2, L),  
             merge( ms(LL), ms(LR) )  
  end.
```

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# Introduce function

```
ms( L ) ->  
  case isBase(L) of  
    true  -> base(L);  
    false -> {LL,LR} = divide(L),  
              merge( ms(LL), ms(LR) )  
  end.
```

```
divide( L ) ->  
  [H|T] = L,  
  {LL,LR} = lists:split(length(L) div 2, L),  
  {LL,LR}.
```

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# Introduce variables

```
ms( L ) ->  
  case isBase(L) of  
    true  -> base(L);  
    false -> {LL,LR} = divide(L),  
              merge( ms(LL), ms(LR) )  
  end.
```

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# Introduce variables

```
ms( L ) ->  
  case isBase(L) of  
    true   -> base(L);  
    false -> {LL,LR} = divide(L),  
              L1 = ms(LL),  
              L2 = ms(LR),  
              merge( L1, L2 )  
  
  end.
```



# Bindings to list

```
ms( L ) ->  
  case isBase(L) of  
    true  -> base(L);  
    false -> {LL,LR} = divide(L),  
              L1 = ms(LL),  
              L2 = ms(LR),  
              merge( L1, L2 )  
  end.
```



# Bindings to list

```
ms( L ) ->  
  case isBase(L) of  
    true  -> base(L);  
    false -> {LL,LR} = divide(L),  
              [L1, L2] = [ms(LL), ms(LR)],  
              merge( L1, L2 )  
  end.
```



# Introduce lists:map/2

```
ms( L ) ->  
  case isBase(L) of  
    true  -> base(L);  
    false -> {LL,LR} = divide(L),  
              [L1, L2] = [ms(LL), ms(LR)],  
              merge( L1, L2 )  
  end.
```



# Introduce lists:map/2

```
ms( L ) ->  
  case isBase(L) of  
    true  -> base(L);  
    false -> {LL,LR} = divide(L),  
              [L1, L2] = lists:map(fun ms/1, [LL,LR]),  
              merge( L1, L2 )  
  
  end.
```

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# Introduce variable

```
ms( L ) ->  
  case isBase(L) of  
    true  -> base(L);  
    false -> {LL,LR} = divide(L),  
             [L1, L2] = lists:map(fun ms/1, [LL,LR]),  
             merge( L1, L2 )  
  end.
```



# Introduce variable

```
ms( L ) ->  
  case isBase(L) of  
    true  -> base(L);  
    false -> {LL,LR} = divide(L),  
              ListOfLists = lists:map(fun ms/1,[LL,LR]),  
              [L1, L2] = ListOfLists,  
              merge( L1, L2 )  
  end.
```



# Introduce function

```
ms( L ) ->  
  case isBase(L) of  
    true  -> base(L);  
    false -> {LL,LR} = divide(L),  
             ListOfLists = lists:map(fun ms/1, [LL,LR]),  
             [L1, L2] = ListOfLists,  
             merge( L1, L2 )  
  end.
```



# Introduce function

```
ms( L ) ->  
  case isBase(L) of  
    true  -> base(L);  
    false -> {LL,LR} = divide(L),  
             ListOfLists = lists:map(fun ms/1, [LL,LR]),  
             combine(ListOfLists)  
  end.
```

```
combine( ListOfLists ) ->  
  [L1, L2] = ListOfLists,  
  merge( L1, L2 ).
```

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# Canonical form

```
ms( L ) ->  
  case isBase(L) of  
    true  -> base(L);  
    false -> {LL,LR} = divide(L),  
             ListOfLists = lists:map(fun ms/1,[LL,LR]),  
             combine(ListOfLists)  
  end.
```

```
combine( ListOfLists ) -> ...  
solve ( L ) -> ...  
isBase( L ) -> ...  
divide( L ) -> ...
```

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# Divide-and-Conquer pattern

```
ms( L ) ->  
  (skel_hlp:dc( fun isBase/1,  
               fun solve/1,  
               fun divide/1,  
               fun combine/1 ))(L).
```

```
combine( ListOfLists ) -> ...  
solve ( L ) -> ...  
isBase( L ) -> ...  
divide( L ) -> ...
```

Kozsik et al.: Refactoring divide-and-conquer functions